Multiobjective Perspective on Online Optimization

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Abstract: Online optimization (OO) addresses iterative and dynamic decision processes under uncertainty and can be viewed as an extension of stochastic optimization. While the latter assumes a priori knowledge of probability distributions of the uncertain variables modeling uncertainty, the fundamental assumption in OO is that the decision's outcome is unknown when the decision is being made. For example, in agriculture, farmers are unaware of the market price of their crops at the time of planting, so it is difficult to maximize revenue with unknown sale prices. This decision situation illustrates an online single-objective optimization (OSO) problem. However, in addition to future market price, they may also consider whether a specific crop replenishes nutrients in the soil or requires higher maintenance costs for water or fertilizer. This now becomes an online multiobjective optimization (OMO) problem as there are multiple conflicting, unknown objective values when the decision to plant crops needs to be made.

While OSO problems have been studied extensively, very few have studied online multiobjective optimization (OMO). We first extend a scalar regret defined for OSO and its bound to a weighted-sum regret in an OMO setting. We next introduce a general concept of regret for OMO problems to recognize that the solutions to multiobjective problems come in the form of efficient sets rather than a single optimal vector, and the solution values come in the form of Pareto sets rather than a single number. The set-based regret assesses the regret for not computing some, or all elements, of the Pareto set of the offline problem. The set-based regret is computed using the hypervolume and is shown to achieve an upper bound that reduces to the bound for the scalar regret. The theoretical results rely only on convexity with no assumption that objective functions follow known distributions or scenarios. Numerical examples are included.