

Syllabus
MATH 9020: Probability Theory II
Spring 2019, Section 001

Instructor: Brian H. Fralix

Instructor's Office: Martin O-310

Telephone Number and Email Address: (864) 656-5197, bfralix@clemson.edu

Office Hours: 2:00PM - 3:00PM MW, and by appointment, Martin O-310.

Course Location and Meeting Time: Martin E-005, 12:20PM - 1:10PM MWF

Course Website: The course website can be found at the following address:

<http://bfralix.people.clemson.edu/math9020.htm>

Textbook: Our primary course textbook will be *A Probability Path* by Sidney Resnick—and we will cover most of Chapters 8, 9, 10. Additional material covered in the course will most likely stem from Billingsley's *Convergence of Probability Measures*, Davidson's *Stochastic Limit Theory*, or Durrett's *Probability: Theory and Examples*.

Ideally, once you finish with MATH 9020, you should be able to pick up a book on e.g. weak convergence theory, stochastic integration and differential equations, point processes, etc., and be comfortable with reading such a book at a reasonable pace. This is the goal I have had in mind while teaching MATH 9010 and 9020.

Prerequisites: MATH 9010. This course is a continuation of MATH 9010, and I will be picking up from where we left off last Fall. We covered all of Chapters 1 through 7 of Resnick last semester. MATH 9020 will begin with Chapter 8 (Weak Convergence), but some of this may be further supplemented with material from Chapter 1 of Billingsley's *Convergence of Probability Measures*, although you will not need to purchase that text.

Attendance Policy: I will keep an official record of your attendance, but your attendance record will not have a direct impact on your grade.

Late Instructor: If the instructor is late, students should wait 15 minutes before leaving.

Course Description: This course covers additional material from measure-theoretic probability theory. Topics include characteristic functions and weak convergence, Central Limit Theorems, Conditional Expectation, Martingale Theory (including the Optional Sampling Theorem and the Martingale Convergence Theorem), plus additional topics from probability theory.

Goals and Objectives: Upon completion of this course, students should be proficient in the following:

- Understanding the role played by characteristic functions in probability theory: both why they are preferred over moment generating functions, and why there is a one-to-one correspondence between characteristic functions and probability distributions on the real line.

- Using characteristic functions to establish convergence in distribution of a sequence of real-valued random variables, and understanding why this approach is valid.
- Using expansions of complex exponentials, combined with characteristic functions to prove various versions of the Central Limit Theorem.
- Understanding the importance of Prohorov's theorem (a collection of probability measures is relatively compact if and only if the collection of measures is tight) and how it is used to justify the use of characteristic functions when establishing convergence in distribution.
- Interpreting conditional expectation with respect to a σ -field as a projection onto a space of appropriately measurable random variables.
- Working with martingales, submartingales, and supermartingales, and understanding their important role in probability theory.
- Understanding how the Optional Sampling Theorem works, as well as some of its applications.
- Understanding the Martingale Convergence Theorem, and its consequences.
- Using Prohorov's theorem to justify weak convergence of probability distributions defined on a σ -field of subsets of a complete, separable metric space.

Grading Policy: Homework will be assigned roughly every 2 weeks throughout the semester. Homework must be turned in on the day it is due.

Your end-of-semester grades will be based on the following scale: $[80, 100] \rightarrow A$, $[65, 80) \rightarrow B$, $[50, 65) \rightarrow C$, and $[0, 50) \rightarrow F$. However, the instructor has the right to "curve upward", i.e. the above scale should be interpreted as "80 and above corresponds to *at least* an *A*", "65 and above corresponds to *at least* a *B*", "50 and above corresponds to *at least* a *C*", and so on.

Homework: Homework assignments will be announced in class, sent to you through email, and posted on the class web site. Your overall grade will be based entirely on homework.

Submitting Homework: Each homework assignment in this course will consist of a collection of problems for you to solve. You are welcome to collaborate with your classmates on the problems, but please state on your solutions anyone you collaborated with while writing up your solutions. You are also free to make use of whatever reference you want while working on each assignment, but please state if you made use of any other resources while writing up your solutions. Furthermore, whenever you begin working on a new problem you should always start on a separate sheet of paper: you will have to state both the problem you have been asked to solve, and then the solution (which of course can be on more than one page, if necessary). **If you do not follow these guidelines on a given assignment, you will receive a grade of zero on that assignment.**

The point of adding this additional structure to the homework assignments is not to penalize you, but to help you realize how important it is to be able to communicate mathematical arguments effectively. If you have not noticed this yet, you will quickly find that taking time to make your arguments clear will improve your understanding of the material.

Official Statement of Academic Integrity: (*From the Graduate Announcements*) “As members of the Clemson University community, we have inherited Thomas Green Clemson’s vision of this institution as a ‘high seminary of learning.’ Fundamental to this vision is a mutual commitment to truthfulness, honor, and responsibility, without which we cannot earn the trust and respect of others. Furthermore, we recognize that academic dishonesty detracts from the value of a Clemson degree. Therefore, we shall not tolerate lying, cheating, or stealing in any form. In instances where academic standards may have been compromised, Clemson University has a responsibility to respond appropriately and expeditiously to charges of violations of academic integrity.””

Disability Access Statement: “Students with disabilities requesting accommodations should make an appointment with Dr. Arlene Stewart (656-6848), Director of Disability Services, to discuss specific needs within the first month of classes. Students should present a Faculty Accommodation Letter from Student Disability Services when they meet with instructors. Accommodations are not retroactive and new Faculty Accommodation Letters must be presented each semester.

Clemson University Title IX Statement: “Clemson University is committed to a policy of equal opportunity for all persons and does not discriminate on the basis of race, color, religion, sex, sexual orientation, gender, pregnancy, national origin, age, disability, veteran’s status, genetic information or protected activity (e.g., opposition to prohibited discrimination or participation in any complaint process, etc.) in employment, educational programs and activities, admissions and financial aid. This includes a prohibition against sexual harassment and sexual violence as mandated by Title IX of the Education Amendments of 1972. This policy is located at

<http://www.clemson.edu/campus-life/campus-services/access/title-ix/>.

Mr. Jerry Knighton is the Clemson University Title IX coordinator. He also is the Director of Access and Equity. His office is located at 111 Holtzendorff Hall, 864-656-3181 (voice) or 864-565-0899 (TDD).”