Mixed Integer Polynomial Optimization - Algorithms and Complexity for Linear and Non-Linear Cases when the Dimension is Fixed

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Abstract: Integer Programming (or Integer Optimization) is the problem of optimizing an objective function over a set of feasible points that have the restriction that some variables must take integer values. Integer linear programming (optimizing a linear function over linear constraints and integer constraints) is NP-Hard, while the continuous counterpart, Linear Programming, is polynomial time solvable. Since the linear case is NP-Hard, the non-linear case of integer programming is also NP-Hard. The story becomes much different when the dimension is considered a fixed number. For instance, what is the complexity of integer linear programming when there are only 3 variables? Lenstra proved in the 80's that in fact, for any fixed dimension, integer linear programming can be solved in polynomial time. This result hinges on the lattice basis reduction algorithms such as the famous LLL algorithm. Surprisingly, the question of complexity in fixed dimension is very unclear even for quadratic integer programming. We will survey Lenstra-type algorithms for integer programming and show recent results on convex and non-convex mixed integer programming.

Bio: Robert Hildebrand is an Assistant Professor at Virginia Tech in the Grado Department of Industrial and Systems Engineering. His research focuses on algorithms and complexity in mixed integer programming. He works on both theory and applied sides of optimization and recent, among other things, is trying to understand difficulties in mixed integer non-linear programming. Robert Hildebrand received his Ph.D. in Applied Mathematics in 2013 from University of California at Davis. His thesis, Algorithms and Cutting Planes for Mixed Integer Programs, was supervised by Professor Matthias Köppe. After graduation, he spent two years in Switzerland as a postdoctoral researcher at the Institute for Operations Research in the Department for Mathematics at ETH Zürich. He was then awarded the Herman Goldstine Memorial Postdoctoral Fellowship at IBM Research for 2015–2017. He started at Virginia Tech in January of 2018.