An Adaptive Discretization Algorithm for the Design of Water Usage and Treatment Networks

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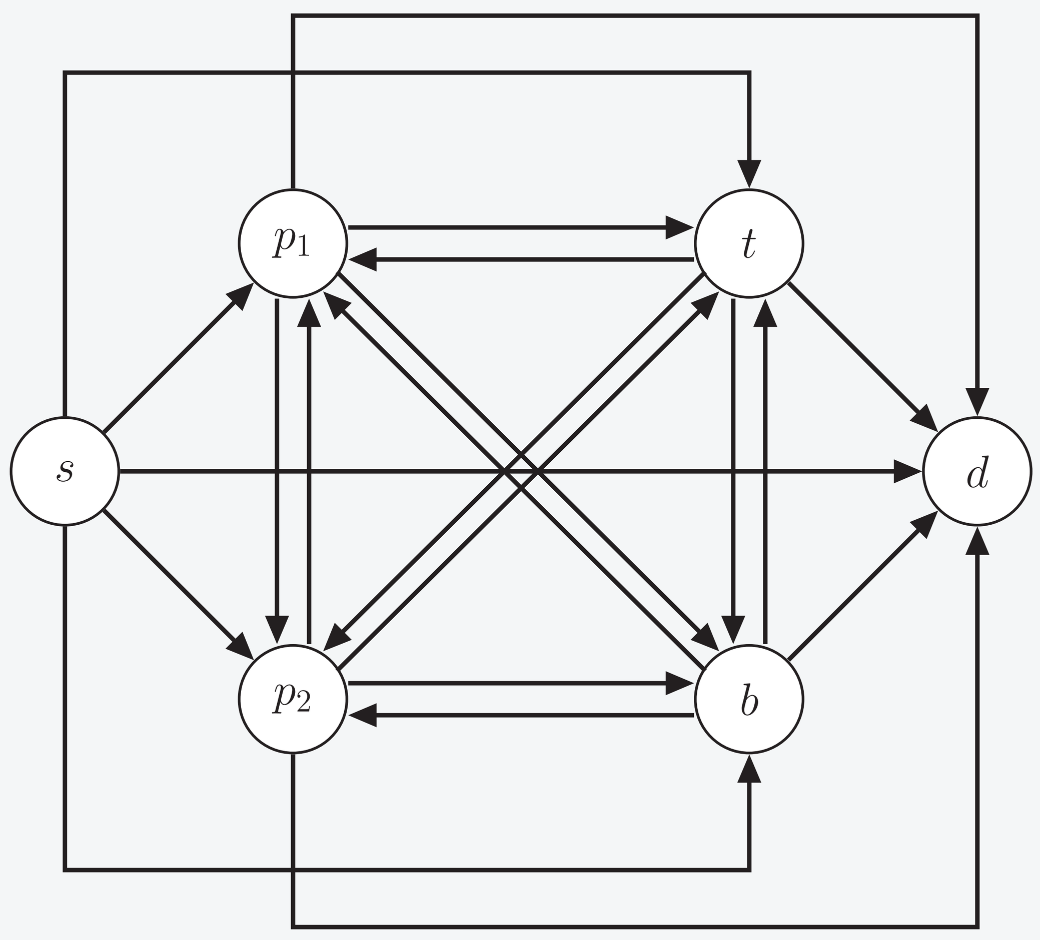
Design of Water Usage and Treatment Networks

- **Water Sources s:** Provide fresh water
- **Process Units p:** Require clean water; Pollute the water during operation
- **Treatment Units t:** Clean the water
- **Buffer Units b:** Store water
- **Water Demands d:** Accept only clean water
- **Decisions:**
  - Build pipes, treatment units, and buffer units
  - Operate system to run process units
- **Objective:** Minimize design and operation costs



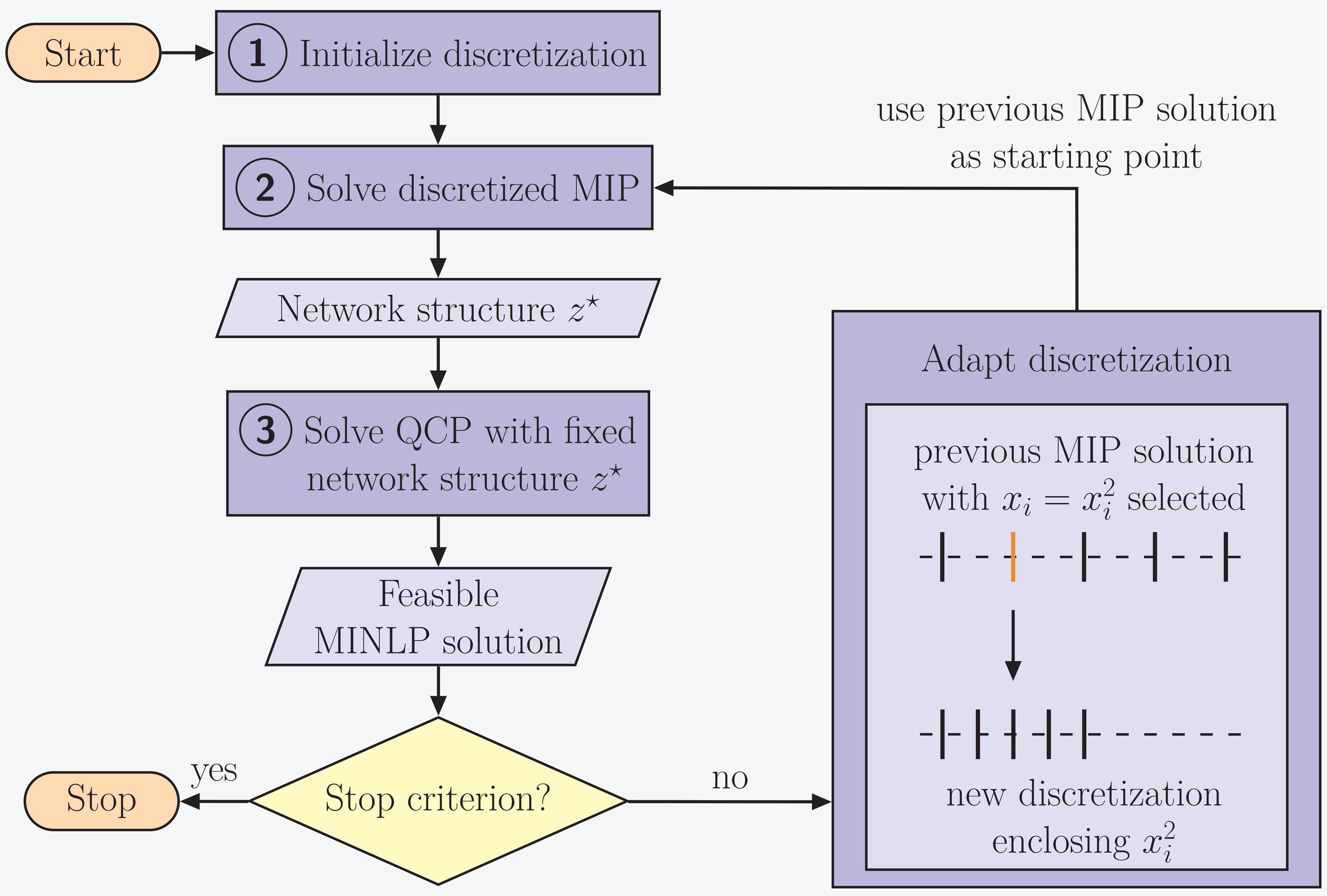
MINLP Formulation

- **Non-convex** MINLP with nonlinear objective and bilinear constraints:

$$\begin{aligned} \min \quad & c_1^\intercal x + c_2^\intercal y + c_3^\intercal z + c_0^\intercal z^\gamma \\ \text{s.t.} \quad & A_1 x + A_2 y + A_3 z \leq b_0 \\ & x^\intercal Q_r y = b_r \qquad \forall r \in \{1, \dots, R\} \\ & x, y \geq 0, \; z \in \mathbb{Z}_+ \end{aligned}$$

- $x \mathrel{\hat=}$ Contaminant concentration
- $y \mathrel{\hat=}$ Flow
- $z \mathrel{\hat=}$ Network structure
- State-of-the-art solvers do not perform well

Adaptive Discretization Algorithm

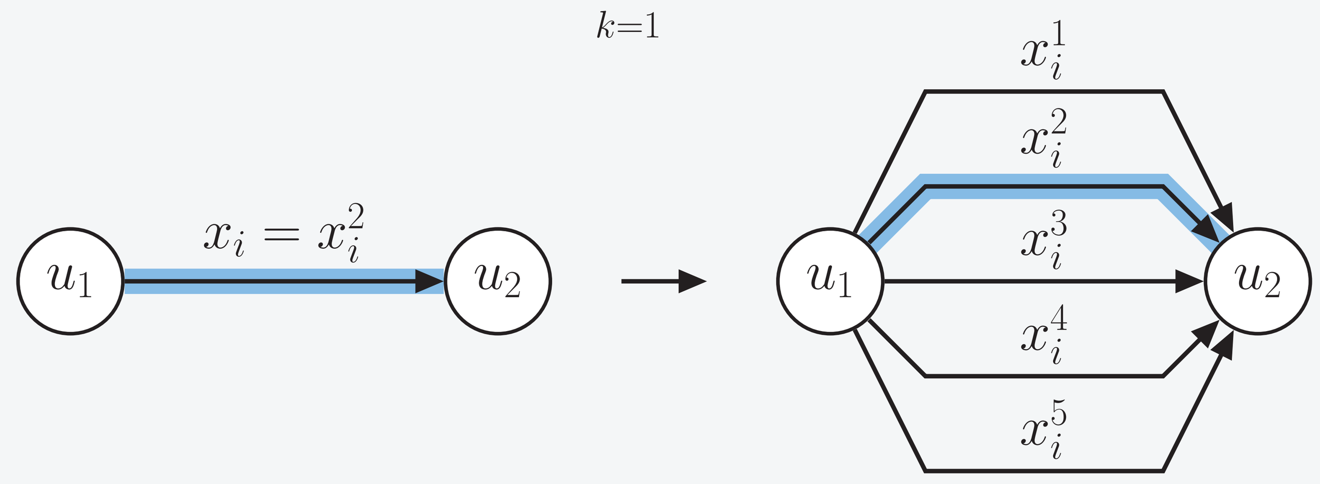


1 Discretize Bilinear Terms

- **Goal:** Eliminate bilinear terms $x_i y_j$
- **Idea:** Discretize x_i as follows:

1. Restrict x_i to K predefined values $x_i^1 < x_i^2 < \dots < x_i^K$
2. Replace pipe by K artificial pipes with flow $y_{j,i}^1, \dots, y_{j,i}^K$
3. Allow only one of these pipes to carry flow via binary variables $b_{j,i}^1, \dots, b_{j,i}^K$
4. Replace bilinear terms by linear ones:

$$x_i y_j = \sum_{k=1}^{K} x_i^k y_{j,i}^k$$



2 Discretized MIP

- Results from MINLP by discretizing bilinear terms and linearizing objective
- Approximates original MINLP
- Usually not feasible to MINLP
- Provides **network structure** $z^\star$

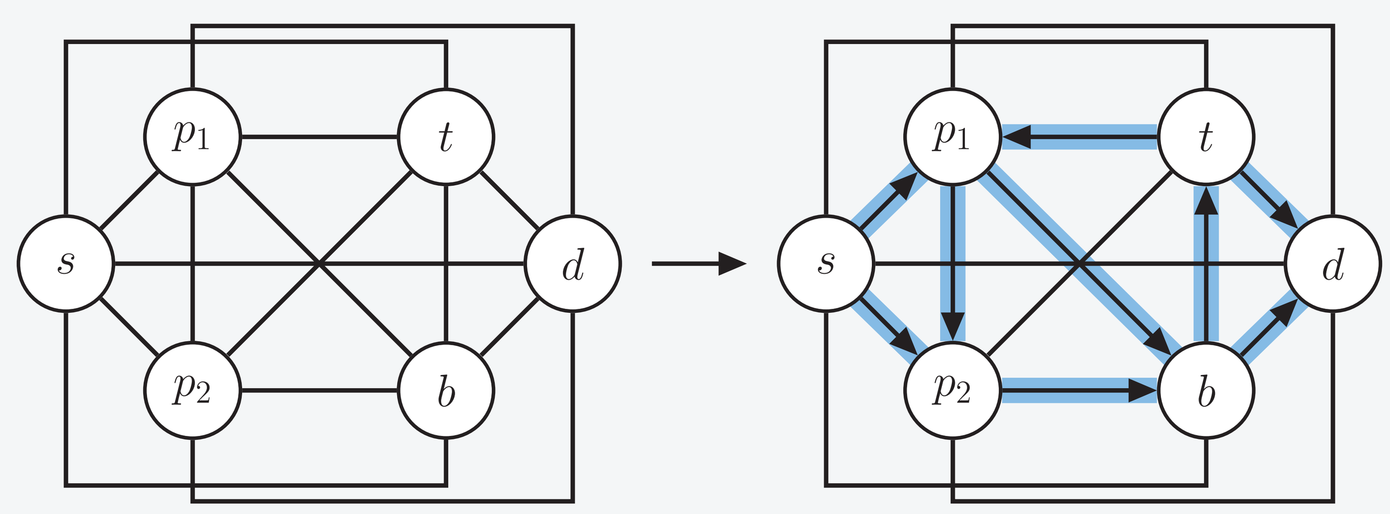
3 QCP

- Results from MINLP by fixing network structure $z^\star$
- Much easier to solve than MINLP
- Provides **feasible** MINLP solution $(x, y, z^\star)$

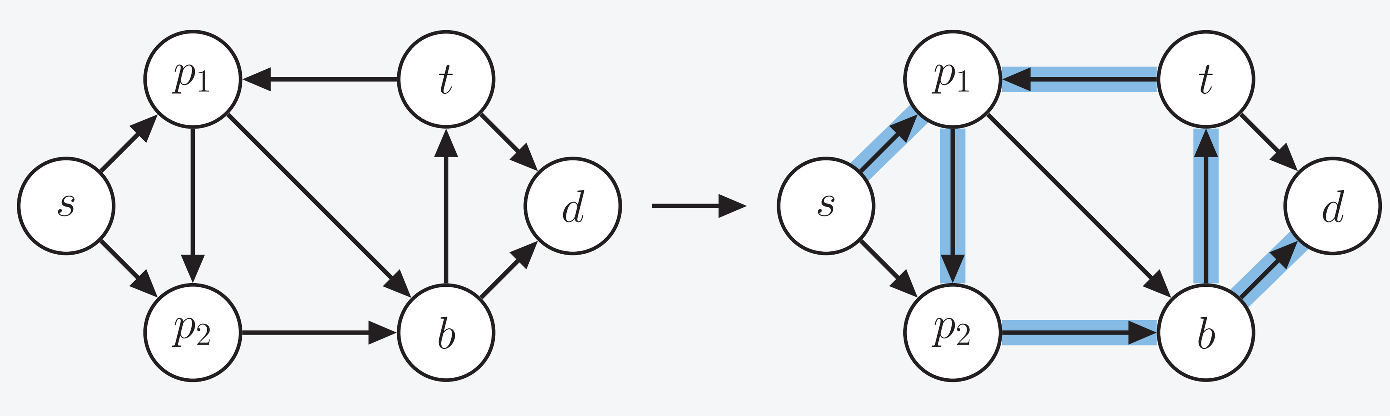
Speeding Up Discretized MIP

- Discretized MIP still hard to solve (many integer variables)
- Solve two subproblems for feasible MIP solution:

1. Fix all pipes at **maximum** size



2. Reduced problem: Consider **only** pipes with positive flow



Computational Results

- 8 randomized real world instances from MINO Challenge (2016)
- MIP solver: CPLEX 12.6.3
- QCP solver: SCIP 3.2
- Objective values are divided by best MINO results
- Algorithm beats MINO results in all but one instance