The Value of Multi-stage Stochastic Programming in Risk-averse Unit Commitment Problems

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Motivation

Unit commitment (UC) is an important and challenging optimization problem in power systems. Variability in net load arising from the increasing penetration of renewable technologies have motivated study of various classes of stochastic UC models.

In **two-stage** (TS) models, the generation schedule for the entire day is fixed while the dispatch is adapted to the uncertainty, whereas in **multi-stage** (MS) models the generation schedule is also allowed to dynamically adapt to the uncertainty realization. Multi-stage models provide more flexibility in the generation schedule, however, they require significantly higher computational effort than two-stage models.

Objective

To justify the additional computational effort to solve the multi-stage models, we provide theoretical and empirical analyses of **the value of multi-stage solution (VMS)** for risk-averse multi-stage stochastic UC models. VMS measures the relative advantage of multi-stage solutions over their two-stage counterparts.

Notation

$t \in \{1, \dots, T\}$: Period index,
$i \in \{1, \dots, n\}$: Generator index,
$u_{it} \in \{0, 1\}$: Status of generator i in period t,
$v_{it} \in \mathbb{R}_+$: Production of generator i in period t,
$w_t \in \mathbb{R}^k$: Auxiliary variables in period t,
$\tilde{d}_t$: Random demand in period t,
X_t : Feasible set in period t,
$f_t(u_{it}, v_{it}, w_t)$: Total cost in period t,
$\rho_t(\cdot)$: One step conditional risk-measure at period t,
$\rho = \rho_2 \circ \rho_3 \cdots \rho_T$ is composite risk measure

Two-stage model

$$
\begin{aligned}
\text{min.}\quad & \rho\left[\sum_{t=1}^{T} f_t(u_t, v_t, w_t)\right] \\
\text{s.t.}\quad & \sum_{i=1}^{n} v_{it} \geq d_t \quad \forall t \\
& \underline{q}_i u_{it} \leq v_{it} \leq \overline{q}_i u_{it} \quad \forall i, t \\
& (u_{t-1}, u_t, v_{t-1}, v_t, w_{t-1}, w_t) \in X_t \quad \forall t \\
& u_t \in \{0,1\}^n, v_t \in \mathbb{R}^n_+, w_t \in \mathbb{R}^k \quad \forall t \\
& [v_t, w_t] = [\tilde{v}_t, \tilde{w}_t](\tilde{d}_{[t]}) \quad \forall t
\end{aligned}
$$

- Easier to solve (+)
- Static assumption on uncertainties (-)

Multi-stage model

$$
\begin{aligned}
\text{min.}\quad & \rho\left[\sum_{t=1}^{T} f_t(u_t, v_t, w_t)\right] \\
\text{s.t.}\quad & \sum_{i=1}^{n} v_{it} \geq d_t \quad \forall t \\
& \underline{q}_i u_{it} \leq v_{it} \leq \overline{q}_i u_{it} \quad \forall i, t \\
& (u_{t-1}, u_t, v_{t-1}, v_t, w_{t-1}, w_t) \in X_t \quad \forall t \\
& u_t \in \{0,1\}^n, v_t \in \mathbb{R}^n_+, w_t \in \mathbb{R}^k \quad \forall t \\
& [u_t, v_t, w_t] = [\tilde{u}_t, \tilde{v}_t, \tilde{w}_t](\tilde{d}_{[t]}) \quad \forall t
\end{aligned}
$$

- Curse of dimensionality, and hence computationally very expensive (-)
- Ability to model dynamic process of uncertainties and decisions (+)

The Value of Multi-stage Solution

The VMS is the difference between the optimal values of TS and MS, that is,

$$\text{VMS} = z^{TS} - z^{MS}$$

where z^{TS} and z^{MS} are the optimal values of TS and MS, respectively.

Bounds on VMS

Assumptions:

- Compete recourse
- No startup/shutdown costs
- Bounded demand: $0 \leq \tilde{d}_t \leq d_t^{max}$ w.p. 1

Main Theorem

$$\alpha_* D^{max} - \alpha^* \rho(\tilde{D}) \leq \ \text{VMS} \ \leq \alpha^* D^{max} - \alpha_* \rho(\tilde{D})$$

where

$\tilde{D} = \sum_{t=1}^{T} \tilde{d}_T$

$D^{max} = \sum_{t=1}^{T} d_T^{max}$

$0 < \alpha_* \leq \alpha^*$ are data dependent constants

Insights

- Suppose $\alpha_* \approx \alpha^* \approx \alpha$
- Then: VMS $\approx \alpha(D^{max} - \rho(\tilde{D}))$
- Note that $0 \leq \rho(\tilde{D}) \leq D^{max}$
- Higher risk aversion increases $\rho(\cdot)$
- Suppose $\tilde{d}_t = \overline{d}_t + \mathcal{U}[-\Delta, \Delta]$ and $\rho[Z] = \mathbb{E}[Z] + \lambda \mathbb{E}\left[(Z - \mathbb{E}[Z])_+\right]$ for some $\lambda \in (0, 1)$, then

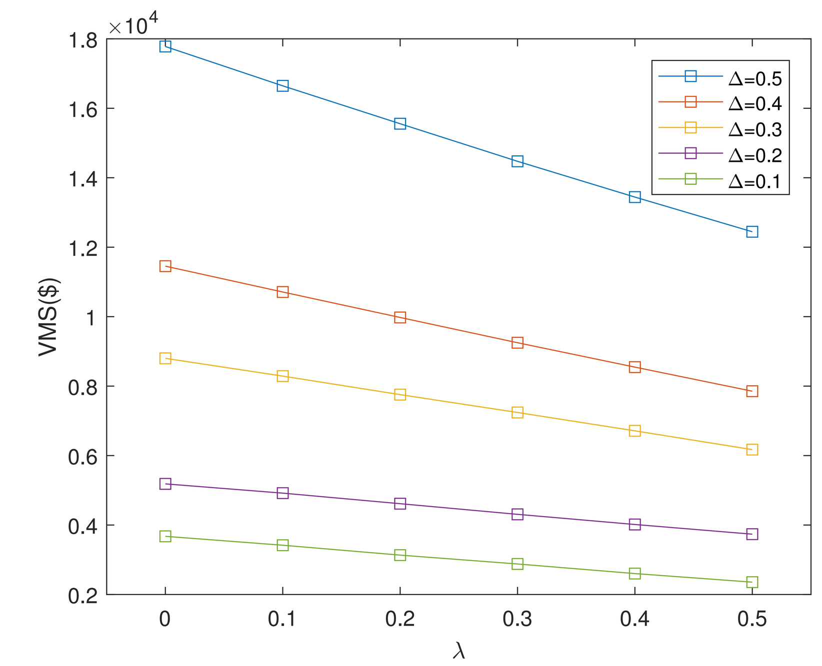
$$\text{VMS} \ \approx \alpha(D^{max} - \rho(\tilde{D})) = \alpha T \left(1 - \frac{\lambda}{4}\right) \Delta$$

Results

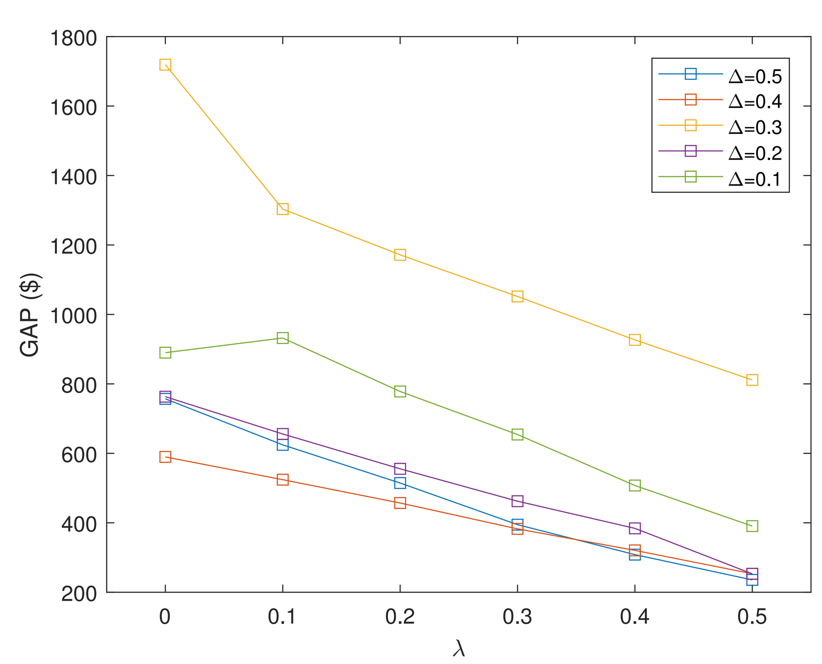
The VMS **increases** with the **number of time periods** (T), with **variability** (Δ) and **decreases** with **risk aversion** (λ).

Numerical illustration

- A 10 generator, 24 period data set from [1] with mean-semideviation risk objective



- Rolling horizon policy obtained by solving TS approximations to the MS problem



Related Literature

[1] S. A. Kazarlis, A. G. Bakirtzis and V. Petridis, "A genetic algorithm solution to the unit commitment problem", *IEEE Transactions on Power Systems*, vol. 11, no. 1, pp. 83-92, 1996.

[2] A. Shapiro, D. Dentcheva and A. Ruszczynski, "Lectures on stochastic programming: modeling and theory", *Society for Industrial and Applied Mathematics*, 2009.

[3] K. Huang and S. Ahmed, "The value of multistage stochastic programming in capacity planning under uncertainty", *Operations Research*, vol. 57, no. 4, pp. 893-904, 2009.