**Introduction**

We consider linear programs with complementarity constraints in the form:

\[
\text{LPCC} \quad \max \ a^T x + b^T y + c^T z \\
\text{s.t.} \quad A x + B y + C z \leq d \\
\quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad y \perp z.
\]

- \( C = \{(y, z) \in \mathbb{R}^2 : y \perp z, 0 \leq y, z \leq 1\} \),
- \( \mathcal{P} = \{(x, y, z) \in \mathbb{R}^{d+z+2} : A x + B y + C z \leq d, 0 \leq x, y, z \leq 1\} \),
- \( \mathcal{F} = \{(x, y, z) \in \mathcal{P} \cap \mathcal{C} = \{(x, y, z) \in \mathbb{R}^{d+z+2} | (A x + B y + C z \leq d, 0 \leq x, y, z \leq 1, y \perp z)\} \).

> We're looking for tighter relaxation for \( \text{conv}(\mathcal{F}) \); \( \text{conv}(\mathcal{G}) \) can be precisely characterized, so need to consider \( \mathcal{P} \) to improve relaxation for \( \mathcal{F} \).

**RLT [1]**

- Add extra variables \( q^i, \bar{q}^i, i \in [n] \) defined by:
  \[
  q^i := \begin{cases} 
  \frac{1}{y^i + z^i} & \text{if } y^i + z^i = 0 \\
  \frac{1}{y^i + z^i} & \text{otherwise}.
  \end{cases}
\]
- Multiply each original constraints with \( q^i, \bar{q}^i \), which gives us new lifted variables:
  \[
  u^i = q^i x^i, \quad v^i = q^i y^i, \quad w^i = q^i z^i \quad \forall i \in [n].
\]
- The complementarity relationship is captured by:
  \[
  w_i^i = 0, \quad \bar{v}^i = 0 \quad \forall i \in [n],
  \]
  \[
  x = u^i + \bar{w}^i, \quad y = v^i + \bar{v}^i, \quad z = w^i + \bar{w}^i \quad \forall i \in [n],
  \]
  \[
  q^i + \bar{q}^i = 1 \quad \forall i \in [n].
\]

**Extended Formulation**

McCormick relaxation on bilinear term (Q):

\[
\begin{align*}
0 \leq & \quad \bar{q}^i - q^i \leq x_j, \quad u_j \geq q^i + x_j - 1, \quad u_j \geq 0, \text{etc.} \\
0 \leq & \quad \bar{u}^i - v^i \leq \bar{q}^i, \quad \bar{v}^i \geq \bar{q}^i + \bar{v}^i - 1, \quad \bar{v}^i \geq 0.
\end{align*}
\]

Multiply each constraint in \( \mathcal{P} \) with \( q^i, \bar{q}^i \) and replace each quadratic term with above variables:

\[
\begin{align}
Au^i + Bu^i + Cw^i & \leq dq^i, \quad Au^i + Bu^i + Cw^i \leq dq^i, \quad \forall i \in [n] \quad (R) \\
0 \leq & \quad u^i, v^i, w^i \leq q^i, \quad 0 \leq u^i, v^i, w^i \leq q^i, \quad \forall i \in [n] \quad (I)
\end{align}
\]

- McCormick relaxation is given by:
  \[
  \mathcal{M} := \{(x, y, z, q, \bar{q}, u, \bar{u}, v, \bar{v}, w, \bar{w}) | (M), (R), (E)\}
\]

**Lemma 0.1**

Denote \( B \) to be the Balas disjunctive formulation in lifted space, then:

\[
B = M;
\]

**Cutting Planes from BQP**

The Boolean Quadric Polytope corresponding to \( G \):

\[
BQP(G) = \text{conv}\{(x, y, z, q, \bar{q}, u, \bar{u}, v, \bar{v}, w, \bar{w}) \in \{0, 1\}^{d+4n+2dn+4n^2} | (Q)\}.
\]

From Proposition 0.2 we can actually have:

\[
\begin{align}
\text{conv}\mathcal{S} & = \text{conv}\{(x, y, z, q, \bar{q}, u, \bar{u}, v, \bar{v}, w, \bar{w}) \in \{0, 1\}^{d+4n+2dn+4n^2} | (Q), (E), (I)\} \\
& = \text{conv}\{(x, y, z, q, \bar{q}, u, \bar{u}, v, \bar{v}, w, \bar{w}) \in \{0, 1\}^{d+4n+4n^2+2dn} | (Q), (E), (I)\}
\end{align}
\]

**Proposition 0.1**

\[
\text{conv}\mathcal{S} = BQP(G) \cap (E).
\]

- Cutting planes from \( BQP(G) \) may be very useful for \( \mathcal{F} \);
- But ONLY cutting planes from \( BQP(G) \) will be useful!

**Future Work**

- Generalize our method to LPCC problems with no specific variable bounds;
- Incorporate special structure on \( B, C \) matrix (like in KKT condition);

**Reference**