Two-stage Stochastic Conic Mixed Integer Programs: Tight Second Stage Formulations

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Two-stage Stochastic Conic Mixed Integer Program (TSS-CMIP)

$$\min \{c^T x + \text{E}[q(x)|\omega] : Ax + b, x \in \mathbb{Z}^n\}$$
where random variable $x_p$ follows a known probability distribution $P$, and for scenario $\omega$ from a finite sample space $\Omega$,

$$\mathbb{E}[q(x)|\omega] := \min \left\{ \sum_{\omega_i \in \Omega} q_{\omega_i}(x) : \mathbb{P}(\omega_i) > 0, x \in \mathbb{R}^n \right\}$$

where $q_{\omega_i}(x)$ are convex conic functions that are tractable.

Structured CMIPs (Atamtürk and Narayanan, 2008)

Theorem 2. Let $\mathbb{E}[q(x)|\omega] = \min \left\{ \sum_{\omega_i \in \Omega} q_{\omega_i}(x) : \mathbb{P}(\omega_i) > 0, x \in \mathbb{R}^n \right\}$

Then, for all $x \in \mathbb{R}^n$, the convex hull of the feasible region of $q(x)|\omega$ for all $x \in \mathbb{R}^n$ is given by

$$\left\{ \begin{array}{l}
\mathbb{E}[q(x)|\omega] \in \mathbb{R}^n \\
\sum_{\omega_i \in \Omega} q_{\omega_i}(x) \\
\mathbb{P}(\omega_i) > 0, x \in \mathbb{R}^n
\end{array} \right\}$$

Second Stage Convexification Approach

Theorem 1. If $\text{conv}([x]) = \{x \in \mathbb{R}^n : Ax + b, x \in \mathbb{Z}^n\}$, then $\text{conv}([x])$ is a partial convex hull of $\mathbb{Z}^n$, where $\mathbb{Z}^n$ has fewer integer constraints (but possibly more linear or nonlinear inequalities) than $\mathbb{Z}^n$, and $\text{conv}([x]) \subseteq \text{conv}([x])$.

Two-Stage Distributionally Robust CMIPs

Our results for TSS-CMIPs are also applicable for the TSDR-CMIP, defined as

$$\min \{c^T x + \text{E}[q(x)|\omega] : Ax + b, x \in \mathbb{Z}^n\}$$
where complete information about the probability distribution $P$ followed by the random variable $x_p$ is not known but it belongs to a set of distributions $\Psi$.

Computational Results

SCMIP $\alpha$, $\beta$, $\gamma$ denotes instance category, where $\alpha$ is the number of integer variables in the first stage, $\beta$ is the number of integer variables in the second stage, and $\gamma$ is the number of mixed-integer constraints in the second stage.

NO-PCUTS: CPLEX 12.70 with its default settings solve the reformulated extensive formulation without our parametric cuts.

WITH-PCUTS: CPLEX 12.70 with its default settings solve the reformulated extensive formulation with our parametric cuts added a priori.

T-EF: time (in seconds).

R: Instances not solved within 3 hours; OM: Out of memory (24 GB RAM).

In Table 1 and Table 2, the first 10 instances are motivated from SLP-LIB instances. The remaining instances have large number of scenarios.

References


2. Rinaldi, M., Zhang, Y. Two-stage stochastic (additionally robust) $\alpha$-order conic mixed integer programs. 18th Integer Programming and Combinatorial Optimization Conference (2017).


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