A mixed-integer approach for the volumetric modulated arc therapy (VMAT) treatment planning problem

Joshua T. Margolis[1], Troy Long[2], and Scott J. Mason[1]

[1]Dept. of Industrial Engineering, Clemson University, Clemson, SC 29634
[2]Radformation Inc., New York, NY 10017

I. Objective

Generate a high-quality, deliverable, desirable-aperture-size VMAT treatment plan given patient geometry and some dosimetric goals that minimizes the deviation between prescribed and administered dose.

II. Introduction

Radiation therapy is one of the main methods of treating cancer. Patients are exposed to internal or external radiation sources with the goals of **1) eradicating cancerous tissue (targets) while sparing healthy structures, or organs-at-risk (OARs)**, and **2) minimizing the chance of future complications**. Treatment planning optimization models are used to construct a set of delivery instructions (treatment plan) to produce a desirable dose distribution when given geometric information about the patient's critical structures, which are discretized into small volumes called voxels. These models set the dose received as a function of radiotherapy machine parameters.

Figure 1: (photo credits to Varian Medical Systems)

III. Literature Review

The following solution methodologies are summarized by Unkelbach *et al.* [2]:

- Intensity modulated arc therapy (IMRT) with Arc Sequencing
- Network algorithms
- Column generation
- Direct machine parameter optimization (DMPO) with local gradient search and trust-region
  - SmartArc (Philips), Oncentra VMAT (Nucleton), RayArc (Raysearch Laboratories), and MONACO (Elektra)
- Simulated annealing
- Conformal arc therapy

IV. Solution Methodology

- Each technique for finding deliverable plan has its own disadvantages
- Preselect set of apertures for each control point
- Allow model to choose successive apertures given feasibility network

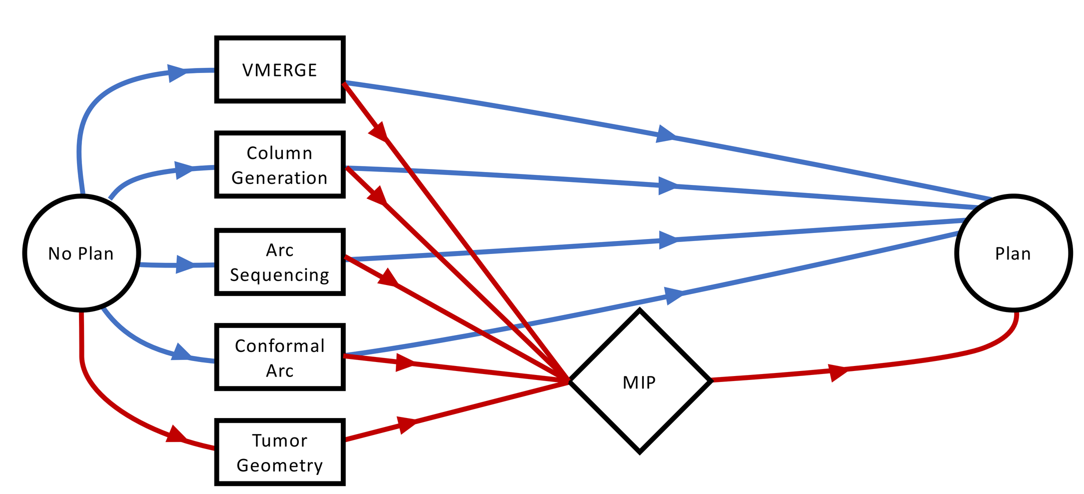

Figure 2: Potential clinical workflow

V. Notation

Sets:

- $\mathcal{V}$ – Set of voxels (discretized internal patient geometry)
- $\mathcal{B}$ – Set of control points (beams)
- $\mathcal{A}_b$ – Subset of apertures for each beam $b \in \mathcal{B}$
- $\mathcal{N}(b,a)$ – Set of apertures at adjacent control points to control point $b \in \mathcal{B}$

Parameters:

- d – Allowable distance leaf positions can move between successive beams given a particular gantry
- $D_{b,a,j}$ – Dose to voxel $j \in \mathcal{V}$ from aperture $a \in \mathcal{A}_b$ at control point $b \in \mathcal{B}$
- $\overline{y}$ – Maximum aperture intensity
- $\underline{y}$ – Minimum aperture intensity
- k – Number of apertures per control point

Decision Variables:

- z_j – Dose at voxel $j \in \mathcal{V}$
- $y_{b,a}$ – Intensity of aperture $a \in \mathcal{A}_b$ at beam $b \in \mathcal{B}$
- $x_{b,a}$ – Indicates if aperture $a \in \mathcal{A}_b$ at beam $b \in \mathcal{B}$ is selected (=1) or not (=0)

VI. Formulation

$$\min_{x,y,z} \quad F(z) \qquad (1)$$

$$\text{s.t. } z_j = \sum_{b \in \mathcal{B}} \sum_{a \in \mathcal{A}_b} D_{b,a,j} y_{b,a} \qquad \forall j \in \mathcal{V} \qquad (2)$$

$$\sum_{a \in \mathcal{A}_b} x_{b,a} = k \qquad \forall b \in \mathcal{B} \qquad (3)$$

$$x_{b,a} + \sum_{a' \in \mathcal{A}_{b+1} \cap \overline{\mathcal{N}}(b,a)} x_{b+1,a'} \leq 1 \qquad \forall b \in \mathcal{B}, a \in \mathcal{A}_b \qquad (4)$$

$$y_{b,a} \leq \overline{y} x_{b,a} \qquad \forall b \in \mathcal{B}, a \in \mathcal{A}_b \qquad (5)$$

$$y_{b,a} \geq \underline{y} x_{b,a} \qquad \forall b \in \mathcal{B}, a \in \mathcal{A}_b \qquad (6)$$

$$x_{b,a} \in \{0,1\} \qquad \forall b \in \mathcal{B}, a \in \mathcal{A}_b \qquad (7)$$

$$z_j \in Z \qquad \forall j \in \mathcal{V} \qquad (8)$$

VII. Proof of Concept

- Model is implemented in Python and solved using Gurobi 7.0.2 on a Mid2014 MacBook Pro
- Utilize The Radiotherapy Optimization Test Set (TROTS) with 23 control points [1]
- Implementation can handle higher resolution
- Not clinically sized cases, but not super small
- Believe solve times to be reasonable

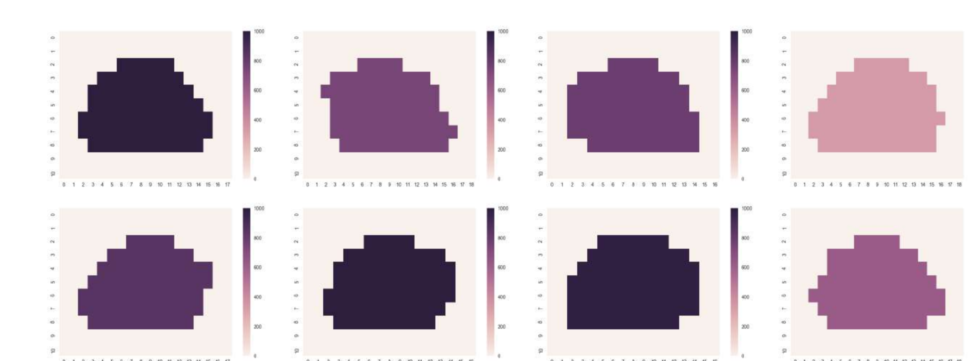

Figure 3: Aperture and intensity for various control points of prostate VMAT plan

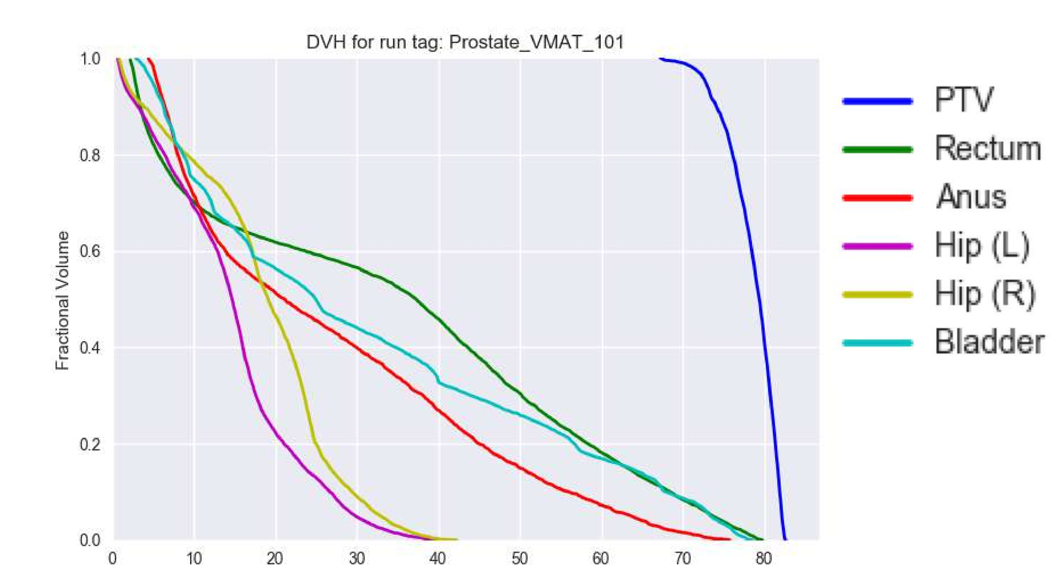

Figure 4: MIP-calculated DVH

VIII. Future Work

- Larger clinical cases
- Extend technique for field-in-field
- Increase size of subset of potential apertures
- Physician independence
- Heuristics/algorithms
- Compare with other solution methodologies

IX. References

[1] S. Breedveld and B. Heijmen, 2017.
[2] J. Unkelbach *et. al.*, 2015.