Mixed Integer Programming Workshop 2018
Clemson University
Greenville, South Carolina

Welcome to Mixed Integer Programming (MIP) 2018, hosted by Clemson University! The MIP workshop series is designed to bring together the integer programming research community in an annual meeting and is organized by volunteer members of the community. We would like to thank our sponsors for their generous support of MIP 2018, which will be the 15th workshop in the series. We look forward to a dynamic and successful workshop and hope that you enjoy your time in Greenville.

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Program for MIP 2018

Monday, June 18		
9:00-9:30	**Registration**	
9:30-9:45	Opening Remarks	
9:45-10:30	Cole Smith	Using binary decision diagrams in solving hard binary optimization problems
10:30-11:00	**Break**	
11:00-11:30	Pierre Le Bodic	Tree-size estimates in MIP solvers
11:30-12:00	Poster teaser I	
12:00-2:00	**Lunch**	
2:00-2:45	Aida Khajavirad	Stronger polyhedral relaxations for polynomial optimization problems
2:45-3:15	Poster teaser II	
3:15-3:45	**Break**	
3:45-4:30	Yuri Faenza	Balas formulation for the union of polytopes is optimal
4:30-5:00	Jim Ostrowski	Almost symmetry in integer programming
5:30-8:00	**Poster session and reception**	

Tuesday, June 19		
9:45-10:30	David Shmoys	Reallocating capacity in bike-sharing systems
10:30-11:00	**Break**	
11:00-11:30	Enrico Malaguti	Integer optimization with penalized fractional values: the knapsack case
11:30-12:00	Ricardo Fukasawa	Split cuts based on sparse disjunctions
12:00-2:00	**Lunch**	
2:00-2:45	Bob Bixby	Progress in MIP solvers with Gurobi
2:45-3:15	Stephen Maher	Large neighborhood Benders' search
3:15-3:45	**Break**	
3:45-4:30	Joseph Paat	Revisiting questions in IP by reparameterizations
4:30-5:00	Quentin Louveaux	The disjunctive hull of facility location problems
6:00-9:00	**Social dinner**	

Wednesday, June 20		
9:45-10:30	Suvrajeet Sen	Cutting plane approximations for stochastic mixed-integer programming
10:30-11:00	**Break**	
11:00-11:30	Marco Lübbecke	Progress in the Branch-Price-and-Cut solver GCG
11:30-12:00	Rob Pratt	Automatic structure detection in mixed integer programs
12:00-2:00	**Lunch**	
2:00-2:45	Annie Raymond	Symmetric sums of squares over k-subset hypercubes
2:45-3:15	Chen Chen	Maximal outer-product-free sets for polynomial optimization
3:15-3:45	**Break**	
3:45-4:30	Wolfram Wieseman	Distributionally robust vechicle routing
4:30-5:00	**Business meeting**	

Thursday, June 21		
9:45-10:30	Gérard Cornuéjols	Cuboids
10:30-11:00	**Break**	
11:00-11:45	Andres Gomez	On connections between mixed-integer fractional and conic quadratic optimization, with applications in best subset selection
11:45-12:15	Miles Lubin	Mixed-integer convex representability
12:15-12:30	**Poster prize**	

Poster Session

Beste Basciftci	Data-driven generator maintenance and operations scheduling under decision-dependent uncertainty
Tim Holzmann	A modified augmented weighted Tchebychev norm for multi-objective combinatorial optimization
Sascha Kuhnke	An adaptive discretization algorithm for the design of water usage and treatment networks
Ben Knueven	Tight generator schedules and unit commitment
Dabeen Lee	Generalized Chvatal-Gomory closures for integer programs with bounds on variables
Haochen Luo	N-step cutset inequalities: facets for multi-module capacitated network design problem
Ali Irfan Mahmutogullari	The value of multi-stage stochastic programming in risk-averse unit commitment problems
Joshua T. Margolis	A mixed-integer model for the volumetric modulated arc therapy treatment planning problem
Hamid Nazari	Optimizing a bundle pricing problem
Prashant Palkar	Mixed-Integer derivative-free optimization
Sriram Sankaranarayanan	Can cut generating functions be good and efficient?
Asteroide Santana	New SOCP relaxation and branching rule for bipartite bilinear programs
Thiago Serra	Bounding and counting linear regions of deep neural networks
Meenarli Sharma	Automatic reformulation techniques for mixed-integer nonlinear programs
Sahar Tahernejad	Valid inequalities for mixed integer bilevel linear optimization problems
Alfredo Torrico	Submodular maximization under matroid constraints: a case for robustness
Eli Towle	External intersection cuts
Hamidreza Validi	A note on "A linear-size zero-one programming model for the minimum spanning tree problem in planar graphs"
Jiawei Wang	Characterization and approximation of strong general dual-feasible functions
Guanyi Wang	Computational evaluation of new dual bounding techniques for sparse PCA
Site Wang	POD.jl: a global MINLP solver based on adaptive piecewise linear relaxations
Zeyang Wu	Optimal sequencing for a pair of square matrices
Jie Zhang	Multi-product newsvendor problem with substitutions: a stochastic integer program perspective
Xuan Zhang	Legal assignments, the EADAM algorithm, and Latin marriages
Yingqiu Zhang	Two-stage stochastic p-order conic mixed-integer programs: tight second-stage formulation
Haoran Zhu	Relaxations for linear programs with complementarity constraints

Abstracts

Monday, June 18:

9:45a-10:30a Cole Smith

Title: Using binary decision diagrams in solving hard binary optimization problems

Abstract: The application of binary decision diagrams (BDDs) in combinatorial optimization has proliferated in the last decade, especially toward binary optimization problems. A BDD is a directed, layered network with one source node and usually two sink nodes (a "true" and a "false" terminal). Every BDD node except for the sink(s) has two outgoing arcs, one corresponding to setting a variable to 0 and the other corresponding to setting that variable to 1, and each arc connects a node in one layer of the BDD to a node in the subsequent layer. The arcs are oriented and weighted in a way that establishes a relationship between the binary optimization problem being modeled and the BDD itself. In particular, there is a correspondence between every source-to-true-sink path in the BDD and a feasible solution (having the same objective) in the binary optimization problem, and every source-to-false-sink path corresponds to an infeasible solution in the binary optimization problem. For problems in which an equivalent and reasonably-sized BDD can be derived, finding an optimal path through the corresponding BDD is an attractive alternative to solving the original binary optimization problem.

In this talk, we also describe how duality information from BDDs can be leveraged in solving special classes of two-stage stochastic integer programming problems with binary variables appearing in both stages. These problems constrain the second-stage variables to belong to the intersection of sets corresponding to first-stage binary variables that equal one. Our approach uncovers strong dual formulations to the second-stage problems by transforming them into dynamic programming (DP) problems parameterized by first-stage variables. We demonstrate how these DPs can be formed by use of binary decision diagrams, which then yield traditional Benders inequalities that can be strengthened based on observations regarding the structure of the underlying DPs. We demonstrate the efficacy of our approach on a set of stochastic traveling salesman problems.

In other settings, though, insisting on the use of a single BDD to model a binary optimization problem is impractical because of the size of the resulting BDD. An alternative approach therefore utilizes not one, but a set of diagrams. Optimizing over a collection of decision diagrams yields a problem called the consistent path problem (CPP), which we can model by associating a network-flow reformulation with each decision diagram and jointly linking decisions through channeling constraints. We investigate the CPP in this talk, including a discussion on complexity results and a thorough polyhedral analysis. We propose a cut-generation algorithm which, under a structured ordering property, finds a cut, if one exists, through an application of the classical maximum flow problem. This procedure is used within a cutting-plane algorithm, which we apply to unconstrained binary cubic optimization and a variant of the market split problem, resulting in a state-of-the-art algorithm for both.

Based on joint work with Leonardo Lozano and David Bergman.

11:00a-11:30a Pierre Le Bodic

Title: Tree-size estimates in MIP solvers

Abstract: We present online methods that estimate the final size of the branch-and-bound tree in Mixed-Integer Programming solvers. This in turn allows to estimate the total running time. One of the method combines an old sampling method due to Knuth (1975) and recent work on branching by Le Bodic and Nemhauser (2017). This method is implemented in the MIP solver SCIP and its results are displayed as an extra column.

2:00p-2:45p Aida Khajavirad
Title: Stronger polyhedral relaxations for polynomial optimization problem
Abstract: We consider the Multilinear set defined by a collection of multilinear terms over the unit hypercube. Such sets appear in factorable reformulations of many types of mixed-integer nonlinear programs including polynomial optimization problems. Utilizing an equivalent hypergraph representation for the Multilinear set, we derive various types of facet defining inequalities for its polyhedral convex hull and present a number of tightness results based on the acyclicity degree of the underlying hypergraph. Subsequently, we detail on the complexity of corresponding separation problems and embed the proposed cut generation algorithm at every node of the branch-and-reduce global solver BARON. Extensive computational results will be presented.

3:45p-4:30p Yuri Faenza
Title: Balas formulation for the union of polytopes is optimal
Abstract: A celebrated theorem of Balas gives an extended formulation for the convex hull P of the union of two polytopes P_1 and P_2. Balas' theorem is used to model disjunctive constraints and in lift-and-project/cutting plane algorithms for (not only) Integer Programming. The number of inequalities in Balas' formulation is linear in the number of facets f_1 of P_1 and f_2 of P_2, and the number of additional variables is linear in the dimension of the space where P_1 and P_2 leave. In this talk, we consider the following question: is there a way to express P in a "cheaper" way? In other words, can we always express P with polynomially many (in $f_1 + f_2$) inequalities, but with a sublinear number of additional variables? We show that this is not possible. Our counterexample is based on the Cayley embedding of suitable perturbations of the polar of the cyclic polytope. We also show that this result essentially carries over if one wants to approximate P and in the more restrictive setting of lift-and-project.

Based on joint work with Michele Conforti and Marco Di Summa.

4:30p-5:00p Jim Ostrowski
Title: Almost symmetry in integer programming
Abstract: Symmetry in integer programming is a reasonably well understood concept. Commercial software has methods that identify and exploit symmetric structure. However, there may be times where an instances symmetric structure is "hidden." Almost symmetric instances are those where one can create symmetric relaxations by making small changes to the instances coefficients. While symmetry-exploiting techniques may not be immediately applicable to such problems directly, they can be used to solve their symmetric relaxations. This talk will focus on how to generate symmetric relaxations and how the quality of the bound they provide compare with the time needed to solve them.

Tuesday, June 19:

9:45a-10:30a David Shmoys
Title: Reallocating capacity in bike-sharing systems
Abstract: The growing popularity of bike-sharing systems around the world has motivated recent attention to models and algorithms for the effective operation of these systems. Most of this

literature focuses on their daily operation for managing asymmetric demand. We shall consider the more strategic question of how to (re-)allocate dock-capacity in such systems. We develop mathematical programming formulations for variations of this problem (service performance over the course of one day, long-run-average performance) and exhibit discrete convex properties in associated optimization problems. This allows us to design a practically fast polynomial-time allocation algorithm to compute optimal solutions for this problem, which can also handle practically motivated constraints, such as a limit on the number of docks moved in the system. We apply our algorithm to data sets from Boston, New York City, and Chicago to investigate how different dock allocations can yield better service in these systems. Recommendations based on our analysis have been adopted by system operators in Boston and New York City. Beyond optimizing for improved quality of service through better allocations, our results also quantify the reduction in rebalancing achievable through strategically reallocating docks

Based on joint work with Daniel Freund and Shane Henderson.

11:00a-11:30a Enrico Malaguti
Title: Integer optimization with penalized fractional values: the knapsack case
Abstract: We consider integer optimization problems where variables can potentially take fractional values, but this occurrence is penalized in the objective function. This general situation has relevant examples in scheduling (preemption), routing (split delivery), cutting and telecommunications, just to mention a few. However, the general case in which variables integrality can be relaxed at cost of introducing a general penalty was not discussed before. As a case study, we consider the possibly simplest combinatorial optimization problem, namely the classical Knapsack Problem. We introduce the Fractional Knapsack Problem with Penalties (FKPP), a variant of the knapsack problem in which items can be split at the expense of a penalty depending on the fractional quantity. We analyze relevant properties of the problem, present alternative mathematical models, and analyze their performance from a theoretical viewpoint. In addition, we introduce a Fully Polynomial Time Approximation Scheme for the approximate solution of the general problem, and an improved dynamic programming approach that computes the optimal solution in one relevant case. We computationally test the proposed models and algorithms on a large set of instances derived from benchmarks from the literature.

Based on joint work with Michele Monaci, Paolo Paronuzzi, and Ulrich Pferschy.

11:30a-12:00p Ricardo Fukasawa
Title: Split cuts from sparse disjunctions
Abstract: Split cuts are arguably the most important types of cuts in the solution of mixed-integer programs (MIP). In addition, sparsity plays an important role in the solution to some MIPs. For instance, the solution of LPs via the simplex method is implemented to take advantage of sparsity. Another example is the recent result of Dey, Molinaro and Wang that relate sparsity of cuts and of MIPs to quality of the final relaxation for certain classes of MIPs.

In this work we exploit sparsity in a different way, namely we investigate how sparse disjunctions affect the quality of the bound obtained using split cuts. Our results indicate that sparse disjunctions can often lead to significant improvements on the quality of the relaxation of MIPs.

Based on joint work with Laurent Poirrier and Shenghao Yang.

2:00p-2:45p Bob Bixby
Title: Progress in MIP solvers with Gurobi
Abstract: To be announced.

2:45p-3:15p Stephen Maher

Title: Large neighborhood Benders' search

Abstract: Enhancements for the Benders' decomposition algorithm can be derived from large neighborhood search (LNS) heuristics. While mixed-integer programming (MIP) solvers are endowed with an array of LNS heuristics, their use is typically limited in bespoke Benders' decomposition implementations. To date, only ad hoc approaches have been developed to enhance the Benders' decomposition algorithm using large neighborhood search techniques—namely local branching and proximity search. A general implementation of Benders' decomposition has been developed within the MIP solver SCIP to permit a greater use of LNS heuristics with the expectation that it will enhance the solution algorithm. Benders' decomposition is employed for all LNS heuristics to improve the quality of the identified solutions and generate additional cuts that can be used to improve the convergence of the main solution algorithm. Focusing on the heuristics of proximity search, RINS and DINS, the results will demonstrate the value of using Benders' decomposition within LNS. We consider some elementary polytopes such as chipped as well as cropped unit hypercubes, and study the performance of some of the strongest lift-and-project operators in computing the convex hull of integral points inside the given elementary polytope. Our study includes the analysis of the number of major iterations required as well as an analysis of the changes in the integrality gaps throughout these major iterations. We will also consider some consequences of these results for MIPs.

Based on joint work with Y. H. (Gary) Au.

3:45p-4:30p Joseph Paat

Title: Revisiting questions in IP by reparameterizations

Abstract: In this talk, we examine new parameterizations of problems in IP, including the Integer Caratheodory number and the proximity of IP and MIP solutions. These reparameterizations lead to new results as well as some interesting connections to group theory and number theory.

Based on joint work with Timm Oertel, Robert Weismantel, and Stefan Weltge.

4:30p-5:00p Quentin Louveaux

Title: The disjunctive hull of facility location problems

Abstract: It is well known that describing explicitly the convex hull of an NP-hard problem is intractable. It is also hard to describe some superset approximations of it like the split or Chvatal closure. Here we investigate the opportunity of describing explicitly some good approximation of the integer hull for some structured problems. In particular, we focus on the disjunctive hull of the facility location problem. We show that it is possible to describe the disjunctive hull in the original space and to separate over it. We also show some initial computational results related to the disjunctive hull.

Based on joint work with Kent Andersen.

Wednesday, May 25:

9:45a-10:30a Suvrajeet Sen

Title: Cutting plane approximations for stochastic mixed-integer programming (SMIP)

Abstract: In this talk we consider the case of SMIP problems with general mixed integer variables (not necessarily all binary). While there have been important advances in SMIP algorithms, all known algorithms today appear to require branch-and-bound procedures to ensure finite convergence of SMIP algorithms. In this paper we address the following question: Is it possible to design an algorithm which provides an optimal solution to an SMIP model with general integer variables (not necessarily all binary) by using cutting planes (and parametric cutting planes) alone. This question is particularly relevant to address whether MIP solutions can be recovered using decomposition algorithms similar to Benders' decomposition. Our study leverages the Cutting Plane Tree (CPT) algorithm which provides finitely convergent cutting planes for general deterministic MIPs.

Based on joint work with Sigme Kúçükyavuz

11:00a-11:30a Marco Lübbecke

Title: Progress in the Branch-Cut-and-Price solver GCG

Abstract: GCG, an extension to the well-known SCIP, is a solver for mixed-integer linear programs. It implements a Dantzig-Wolfe (or similar) reformulation and a full-featured branch-price-and-cut algorithm. Information on how the reformulation should be performed can be given by the user in various ways. However, GCG can and usually does detect a model structure suited for reformulation all by itself. We report on recent developments that lead to the upcoming release 3.0. This includes a completely re-designed structure detection, new cutting planes, and experimental features like deciding whether a reformulation should be applied at all and a Benders decomposition extension. We show computational experiments and some use cases in which we applied GCG.

Based on joint work with Michael Bastubbe, Stephen Maher, Christian Puchert, Jonas Witt.

11:30a-12:00p Rob Pratt

Title: Automatic structure detection in mixed-integer programming

Abstract: Certain classes of mixed-integer programs are known to be efficiently solvable by exploiting special structures embedded in their constraint matrices. One such structure is the bordered block diagonal (BBD) structure that lends itself to Dantzig-Wolfe reformulation (DWR) and branch-and-price. Given a BBD structure for the constraint matrix of a general MIP, several platforms (such as COIN/DIP, SCIP/GCG and SAS/DECOMP) exist that can perform automatic DWR of the problem and solve the MIP using branch-and-price. The challenge of using branch-and-price as a general-purpose solver, however, lies in the requirement of the knowledge of a structure a priori. We propose a new algorithm to automatically detect BBD structures inherent in a matrix. We start by introducing a new measure of goodness to capture desired features in BBD structures such as granularity of the structure, homogeneity of the block sizes, and isomorphism of the blocks. The main building block of the proposed approach is the modularity-based community detection in lieu of graph/hypergraph partitioning methods to alleviate one major drawback of the existing approaches in the literature: predefining the number of blocks. When tested on MIPLIB (and customer) instances using the SAS/DECOMP framework, the proposed algorithm was found to identify structures that, on average, lead to significant improvements both in computation time and optimality gap compared to those detected by the state-of-the-art BBD detection techniques in the literature.

Based on joint work with Taghi Khaniyev, Samir Elhedhli. (Note: Rob is kindly giving this presentation in lieu of Matthew Galati.)

2:00p-2:45p Annie Raymond

Title: Symmetric sums of squares over k-subset hypercubes

Abstract: Polynomial optimization over hypercubes has important applications in combinatorial optimization. We develop a symmetry-reduction method that finds sums of squares certificates for non-negative symmetric polynomials over k-subset hypercubes that improves on a technique due to Gatermann and Parrilo. For every symmetric polynomial that has a sos expression of a fixed degree, our method finds a succinct sos expression whose size depends only on the degree and not on the number of variables. Our results relate naturally to Razborov's flag algebra calculus for solving problems in extremal combinatorics. This leads to new results involving flags and their power in finding sos certificates.

Based on joint work with James Saunderson, Mohit Singh and Rekha Thomas.

2:45p-3:15p Chen Chen
Title: Maximal outer-product-free sets for polynomial optimization
Abstract: We consider the application of S-free sets, or convex forbidden zones, to polynomial optimization; such sets are given the name outer-product-free sets in this special application. Deeper intersection cuts are derived from larger S-free sets, and so there is a natural emphasis in the literature on (inclusion-wise) maximal S-free sets. Maximal S-free sets are essential for deriving general-purpose valid inequalities for mixed-integer linear and (more recently) mixed-integer conic programming; such inequalities, namely split cuts, are indispensable to modern branch-and-cut solvers. Our work focuses on developing analogous cuts for polynomial optimization. We introduce two infinite families of maximal outer-product-free sets: one which recovers a standard semidefinite programming relaxation, and another which is related to a characterization of a matrix rank-one condition. This talk also includes discussion about implementation in MINLP branch-and-cut software.

3:45p-4:30p Wolfram Wieseman
Title: Distributionally robust vehicle routing
Abstract: We study a variant of the capacitated vehicle routing problem (CVRP), which asks for the cost-optimal delivery of a single product to geographically dispersed customers through a fleet of capacity-constrained vehicles. Contrary to the classical CVRP, which assumes that the customer demands are deterministic, we model the demands as a random vector whose distribution is only known to belong to an ambiguity set. Moreover, we require the delivery schedule to be feasible with a probability of at least $1 - \epsilon$, where ϵ characterizes the risk tolerance of the decision maker. We argue that the emerging distributionally robust CVRP can be solved efficiently with modern branch-and-cut algorithms if and only if the ambiguity set satisfies a subadditivity condition. We then show that this subadditivity condition holds for a large class of moment ambiguity sets. We derive efficient cut generation schemes for ambiguity sets that specify the support as well as (bounds on) the first and second moments of the customer demands. Our numerical results indicate that the distributionally robust CVRP has favorable scaling properties and can often be solved in runtimes comparable to those of the deterministic CVRP.

Thursday, June 21:

9:45a-10:30a Gérard Cornuéjols
Title: Cuboids
Abstract: The $\tau = 2$ Conjecture, the Replication Conjecture and the f-Flowing Conjecture, and the classification of binary matroids with the sums of circuits property are foundational to Clutter Theory and have far-reaching consequences in Combinatorial Optimization, Matroid Theory and Graph Theory. We prove that these conjectures and result can equivalently be formulated in terms of cuboids, which form a special class of clutters. Cuboids are used as means to (a) manifest the geometry behind primal integrality and dual integrality of set covering linear programs, and

(b) reveal a geometric rift between these two properties, in turn explaining why primal integrality does not imply dual integrality for set covering linear programs. Along the way, we see that the geometry supports the $\tau = 2$ Conjecture. Studying the geometry also leads to over 700 new ideal minimally non-packing clutters over at most 14 elements, a surprising revelation as there was once thought to be only one such clutter.

Based on joint work with Ahmad Abdi, Natalia Guricanova and Dabeen Lee.

11:00a-11:45a Andres Gomez
Title: On connections between mixed-integer fractional and conic quadratic optimization, with applications in best subset selection
Abstract: Mixed-integer fractional optimization and mixed-integer conic quadratic optimization have, for the most part, been studied independently. However, fractional optimization problems can often be formulated as conic quadratic optimization problems and vice versa. Thus, new algorithmic approaches for both classes of problems can be developed by exploiting these connections. In this talk we briefly review some recent results that use the link between fractional and second-order conic optimization. Then, we discuss in detail an application in statistics. In particular, we show that the problem of selecting the best linear regression model according to a broad class of well-known information criteria can be modeled as mixed-integer fractional/conic quadratic optimization. We show how to strengthen the formulations by using convexity/concavity in the objective function, and by exploiting the structure of the optimal solutions. We also propose a variant of the Newton method for fractional optimization to solve the resulting MINLPs. We show that the proposed approach outperforms alternatives proposed in the literature. We also outline how the ideas presented in the talk can be used in other contexts beyond best subset selection.

11:45a-12:15p Miles Lubin
Title: Mixed-integer convex representability
Abstract: Motivated by recent advances in computational mixed-integer convex programming (MICP), we consider the question of which nonconvex sets can be represented exactly as the projections of MICP feasible regions. Our answers help clarify the scope and limitations of MICP. We completely characterize the mixed-binary case and develop necessary conditions for the general mixed-integer case. We are able to show, for example, that the set of prime numbers is not MICP representable, although many irregular sets are. We prove, however, that under a rationality condition, MICP representable sets satisfy similar periodicity conditions as mixed-integer linear (MILP) representable sets.

Based on joint work with Ilias Zadik and Juan Pablo Vielma.

POSTER SESSION ABSTRACTS

Beste Basciftci, Shabbir Ahmed, Nagi Gebraeel

Title: Data-driven generator maintenance and operations scheduling under decision-dependent uncertainty

Extended abstract: Constructing reliable and cost-effective maintenance schedules is an important concern in power systems. Operational decisions, such as on-off statuses of generators and their dispatch amount, play a pivotal role in this regard as higher (lower) loads result in an accelerated (deaccelerated) degradation process, which may require scheduling maintenance at earlier (later) time. Despite the tight coupling between generator conditions and operational decisions, integrating this into maintenance optimization has not been explored in detail. We propose a novel optimization framework that determines maintenance and operations schedules of generators with explicit consideration of load-dependent degradation and uncertainty of generator failures.

We model the load-dependent generator maintenance and operations scheduling problem as a stochastic program. We propose a chance-constraint restricting the number of generators that fails before their scheduled maintenance with a high probability. The remaining constraints of the optimization model enforce maintenance restrictions and operational level constraints such as demand satisfaction, production capacities, transmission line limits. To capture the endogenous effect of the operational decisions, we propose a data-driven degradation modeling framework that considers unexpected failure times of the generators. The maintenance cost function and chance-constraint leverage this framework by considering the estimated remaining lifetime distributions of the generators. Thus, they involve decision-dependent uncertainties, as the failure probabilities depend on dispatch and commitment decisions. To represent the chance-constraint, we extend a safe approximation proposed in the literature[1] to the decision-dependent setting. As the resulting mathematical program includes nonlinear terms in the objective and chance-constraint due to the structure of the cumulative distribution function of remaining lifetime, we propose a two-step procedure. First, we map the continuous degradation amount of the generators to the maintenance cost and the remaining lifetime distribution functions by a piecewise linearization approach. Since these functions are not convex, we adopt the log formulation[2]. Secondly, we utilize McCormick envelopes for linearizing the bilinear terms. To obtain an even stronger formulation, we improve the upper bounds on the variables and conduct a polyhedral study by taking into account the special structure of the problem.

We demonstrate the effectiveness of the proposed approach with a decision-dependent simulation framework. We assess load-dependent solutions over a set of scenarios, and compare the unexpected failures and the associated costs with that of the load-independent and reliability based models. We illustrate our results on three IEEE instances, 9-bus, 39-bus, and 118-bus. We present our computational study under different system congestion and precaution levels over various maintenance coefficients of objective. Our computational experiments demonstrate the significant cost savings and reductions in failures illustrating the importance of considering operational decisions in condition-based maintenance scheduling. Our results also highlight runtime improvements due to the proposed formulation enhancements together with a branching strategy that prioritizes the maintenance decisions over commitment decisions.

[1] B. Basciftci, S. Ahmed, N. Gebraeel, M. Yildirim, "Stochastic Optimization of Maintenance and Operations Schedules under Unexpected Failures", *IEEE Transactions on Power Systems, 2018.*

[2] J. P. Vielma, S. Ahmed, G. Nemhauser, "Mixed-integer models for nonseparable piecewise-linear optimization: Unifying framework and extensions", *Operations Research, 2010.*

Sanjeeb Dash, Oktay Günlük, **Dabeen Lee**

Title: Generalized Chvátal-Gomory closures for integer programs with bounds on variables

Extended abstract: Integer programming problems often have nonnegative or bounded variables. Chvátal-Gomory (CG) cuts form a practically useful class of cutting planes for integer programs, but usually do not incorporate all available variable bounds. We consider a natural generalization of CG cuts that uses available bound information, studied earlier by Dunkel and Schulz (2011), and by Pokutta (2011). We prove that the closure of a rational polyhedron, defined as the set of points satisfying all the generalized CG cuts, is also a rational polyhedron.

We consider a rational polyhedron $P \subseteq \mathbb{R}^n$ where the points in $P \cap \mathbb{Z}^n$ are contained in some proper subset S of $\mathbb{Z}^n$. In particular, we consider $S = \{x \in \mathbb{Z}^n : \ \ell \leq x \leq u\}$ for some $\ell, u \in (\mathbb{R} \cup \{\pm\infty\})^n$. Given an inequality $cx \leq d$ valid for P, with integral coefficients $c \in \mathbb{Z}^n$ that is valid for P, $cx \leq \max\{cz : \ cz \leq d, \ z \in S\}$ is still valid for all $z \in P \cap \mathbb{Z}^n$, and this inequality dominates $cx \leq \lfloor d \rfloor$. We call $cx \leq \max\{cz : \ cz \leq d, \ z \in S\}$ the *S-Chvátal-Gomory cut* obtained from $cx \leq d$. Based on this generalization of CG-cuts, we can also generalize the notion of the Chvátal-Gomory closure. For a rational polyhedron $P \subseteq \mathbb{R}^n$ and $S \subseteq \mathbb{Z}^n$ containing $P \cap \mathbb{Z}^n$, the *S-Chvátal-Gomory closure* of P is defined as

$$C_S(P) := \bigcap_{\pi \in \mathbb{Z}^n} \left\{ x \in \mathbb{R}^n : \ \pi x \leq \max \left\{ \pi z : \ \pi z \leq \max_{y \in P} \pi y, \ z \in S \right\} \right\}.$$

In words, the S-Chvátal-Gomory closure of P is obtained by applying all possible S-Chvátal-Gomory inequalities.

The Chvátal closure of a rational polyhedron is polyhedral, meaning that it is described by finitely many inequalities. A natural question we can ask is whether the S-Chvátal closure of a rational polyhedron is also a rational polyhedron. Dunkel and Schulz (2011) proved that $C_S(P)$ is a rational polytope when $S = \{0, 1\}^n$ and $P \subseteq [0, 1]^n$ is a rational polytope. We extend this result to the case where S is the set of integer points that satisfy any arbitrary variable bounds. Namely, we prove that $C_S(P)$ is a rational polyhedron if $S = \{x \in \mathbb{Z}^n : \ \ell \leq x \leq u\}$ for some $\ell, u \in (\mathbb{R} \cup \{\pm\infty\})^n$ and P is a rational polyhedron contained in $\mathrm{conv}(S)$.

Tim Holzmann, J. Cole Smith

Title: A Modified Augmented Weighted Tchebychev Norm for Multi-Objective Combinatorial Optimization

Extended abstract: In the real-world, decision makers' must struggle with uncertainty. Common approaches to uncertain mixed integer programs include stochastic optimization and robust optimization. Klamroth et al. (2017) suggest an appoach using multi-objective optimization, wherein they create a separate objective for each realization of the uncertain parameter and then seek to identify non-dominated solutions. However, such a strategy requires a multi-objective generating algorithm which is robust for a larger number of dimensions. Leading approaches in the literature for generating the full set of non-dominated solutions to multi-objective problems include ϵ-constraint methods, two-phase methods, branch and bound methods, and Klein and Hannan's family of methods. Most applications of these methods, however, focus on only two or three objectives.

This poster will present the Modified Augmented Weighted Tchebychev (MAWT) norm. The MAWT norm is similar to Steuer and Choo's augmented weighted Tchebychev norm approach, except that we apply the weighting scheme to both the base norm and the augmentation term. With this simple adjustment, we are able to afford a simpler method to identify the right coefficients for the augmentation term which both guarantee that only non-dominated solutions are returned and also that no non-dominated solutions are missed. Moreover, unlike Dächert, Gorski, and Klamroth's (2012) methods of parameter selection for the augmented weighted Tchebychev norm, our approach applies for any number of objectives. Thus, to our knowledge, this is the first algorithm that uses a variant of the Tchebychev norm to generate the non-dominated set of solutions for multi-objective combinatorial optimization problems.

To accompany the norm, we give an algorithm that, under very general assumptions, uses the MAWT norm to generate the non-dominated set for a multi-objective combinatorial optimization problems. Finally, we present a computational study wherein we compare two variants of our MAWT algorithm against two ϵ-constraint algorithms recently published by Kirlik and Sayïn (2014) and by Özlen, Burton, and MacRae (2014). Our study uses a testbed of problems with 3–6 objectives, 25–100 decision variables, and non-dominated sets ranging from 2–1,378 points. Our study indicates that the MAWT norm renders more complex integer programming sub-problems than the competing algorithms (which use a lexicographic preference structure). Nevertheless, as the number of objectives increases, we find that our algorithms based on the MAWT norm are able to out-perform the competing algorithms, generating the non-dominated set of solutions an order of magnitude faster.

Arie M.C.A. Koster, **Sascha Kuhnke**

Title: An Adaptive Discretization Algorithm for the Design of Water Usage and Treatment Networks

Extended abstract: Water is extensively used in many different processes in industrial plants such as oil refineries or chemical plants. Due to scarcity of good industrial water and increasing environmental requirements for wastewater, effective wastewater treatment and water reuse is essential. This motivates the design of water usage and treatment networks which contain both water using units and wastewater treatment units. In such networks, the water using units have to be supplied with clean water and environmental regulations for wastewater have to be met. To remove contaminants from the water, wastewater treatment units can be installed and operated. The objective of the design problem is to simultaneously optimize the network structure and water allocation of the system at minimum total cost.

Due to many bilinear mass balance constraints, this water allocation problem is a nonconvex mixed integer nonlinear program (MINLP) where nonlinear state-of-the-art solvers such as BARON or SCIP have difficulties to find feasible solutions for real world instances. These difficulties arise for two reasons. On the one hand, the formulation contains numerous nonconvex bilinear constraints and, on the other hand, the program features many integer variables. Therefore, we introduce an iterative algorithm to solve this problem by treating these issues separately. In each iteration, a feasible solution to the original problem is calculated via an interaction of an MILP and a quadratically constrained program (QCP). First, the algorithm solves an MILP which approximates the original MINLP by discretizing contaminant concentrations. In this discretized MILP, all bilinear terms are eliminated and its solution yields a suitable network structure. Then, by fixing this structure, all integer variables of the original problem turn into parameters. We obtain a QCP which provides feasible solutions to the original MINLP. To generate more accurate structures which lead to better solutions of the original problem, the discretization of the MILP is adapted after each iteration based on the previous MILP solution. In many cases where nonlinear solvers fail, our approach leads to feasible solutions with good solution quality in short running time.

Even though all bilinear terms are eliminated in the discretized MILP, it contains a large number of integer variables since many binary variables have been added to discretize the bilinear terms. Hence, solving the discretized problem could still take too much time for larger instances. As we need to solve the discretized MILP in each iteration of the algorithm, a faster running time for this problem is desired. Therefore, we developed the Max-Pipes Heuristic which consists of two phases to obtain good solutions for the discretized MILP in shorter running time. In the first phase, a simplified version of the discretized problem where all pipes are built at maximum size is solved to rule out decisions which are unlikely in an optimal solution. In the second phase, a feasible MILP solution is determined by only considering the reduced solution space. This heuristic improved the performance of the whole algorithm significantly as the running times are faster and even the objective values are improved.

We conducted a computational study to analyze the performance of the Adaptive Discretization Algorithm where we compared our results to the best solvers submitted in the MINO industrial challenge (2016). Here, our algorithm was able to achieve better objective values within the given time limit than the best results from the challenge in almost all cases.

Ben Knueven, Jim Ostrowski, Jianhui Wang, and Jean-Paul Watson

Title: Tight Generator Schedules and Unit Commitment

Extended abstract: The unit commitment problem (UC) is that of scheduling power generating units to meet anticipated energy demand at minimal cost, while meeting the physical constraints of the generating units. In the past decade and a half, all system operators in the United States have switched from using Lagrangian relaxation to using MIP to solve UC, with a savings estimated at $5 billion per year. Therefore, improvements to MIP models for UC are of both practical and academic interest.

This work will present some recent results on MIP models for UC. First, we give a convex hull description of the feasible operating schedule of a generator. Moreover, this description is fairly flexible and can enable a variety of additional physical constraints. Second, we give a tighter description of time-dependent startup costs for a single generator. Last, we demonstrate how both of these formulations can be used to exploit symmetry in UC via generator aggregation.

The convex hull description is inspired by a dynamic programming model laid out by Frangioni and Gentile for a ramping-constrained generator. The formulation developed can be used to model any properties of a generator that are polyhedrally representable when the commitment status is fixed. To prove the integrality of this formulation we extend Balas' classical result on unions of polyhedra to allow for arbitrary constrained sums of polyhedra. Unfortunately, this extended formulation is large as it requires many copies of the generator's production variable, and is therefore unsuited (in most cases) for direct use in a unit commitment model. To circumvent this we propose a cut-generating linear program for UC which, for each generator, checks if its relaxation is in the generator's convex hull, and if not returns a valid cut.

The second part of this work is a novel matching formulation for time-dependent startup costs for a generator. Thermal generating units cool slowly, and if they are turned on again in short order, they have a lower startup cost. The matching formulation we propose is larger than existing formulations in the literature, but smaller than the obvious perfect formulation. However, the proposed matching formulation is practically tight – under reasonable assumptions, it has the same LP relaxation value as the perfect formulation. When compared against existing formulations, the proposed matching formulation is generally best computationally, especially on UC problems with more variation in net-load due to renewables.

The final piece of this work is demonstrating how the tighter formulations given before can be used to exploit symmetry in unit commitment. Because generators are only bound together by the energy demand (and possibly reserve) at each time step, interchanging two generators' schedules is feasible so long as the two generators have identical parameters. Using the tight formulations described above, we show that under certain conditions the production of identical generators can be represented as an aggregated generator, reducing the size of the model. For example, if $u_t^1, \ldots, u_t^n$ are the binary on/off variables for identical generators $1, \ldots, n$ at time t, we can consider $U_t \in \{0, 1, \ldots, n\}$ to represent *how many* of the n identical generators on are on at t. Under conditions we present and using the formulations mentioned above, such a reduction can be done exactly. Some simple post-processing allows us to recover an optimal solution to the original problem.

Kiavash Kianfar, **Haochen Luo**

Title: n-step cutset inequalities: facets for the multi-module capacitated network design problem

Extended abstract: In this work, we consider the *multi-module capacitated network design* (MMND) problem, where minimum cost flows as well as flow transfer capacities are to be decided on a network such that the supply of some vertices is transferred through the network to satisfy the demand of some other vertices. The objective is to minimize the cost of installing capacities and transferring flows. Such problem arises in many applications, especially in telecommunication where traffic is transferred along the telecommunication network through transmission facilities like fiber-optic cables. In real-world scenarios, such transmission facilities are often available in different sizes, or modules, for purchase or installation. Each module can be any non-negative constant and a fixed-charge cost is associated with each module. Decision-making becomes more complicated in this case since decisions need to be made on each of the capacity modules.

In the literature, mixed integer programming (MIP) formulations of similar network design problems have been broadly studied. A considerable number of these studies have addressed MMND from the cutting plane perspective. Among various types of valid inequalities for network design problems, the most effective one is obtained through the idea of partitioning the set of vertices. We refer to this type of valid inequalities as cutset inequalities. Magnanti and Mirchandani (1993) considered special cases of MMND with 1,2, and 3 divisible capacity modules and provided cutset facets in the space of capacity variables. Bienstock and Günlük (1996) studied MMND with 2 divisible capacity modules and preinstalled capacity on edges, and introduced the class of flow cutset facets. Chopra et al. (1998) studied MMND with 1 and 2 divisible capacity modules and developed a more general class of cutset inequalities we refer to as general cutset inequalities. They showed that MMND is NP-hard even with only 1 capacity module. Atamtürk (2002) provided the class of multifacility cutset inequalities for MMND by studying the polyhedral structure of the cutset polyhedron, a mixed integer set obtained from relaxing and aggregating the constraints of MMND. Raack at al. (2011) further studied the cutset polyhedron and showed that facets for cutset polyhedron are also facets for MMND under certain conditions.

It can be shown that the aforementioned inequalities for MMND can be obtained by mixed integer rounding (MIR). Kianfar and Fathi (2009) generalized the idea of MIR to n-step MIR facets for the mixed integer knapsack set. Based on n-step MIR, we introduce a new class of valid inequalities for MMND, referred to as *n-step cutset inequalities*. We show that n-step cutset inequalities contain cutset inequalities in the literature as special cases. We then prove that n-step cutset inequalities are facet-defining for the convex hull of the cutset polyhedron, and further also facet-defining for the MIP formulation of MMND under certain conditions, and therefore provide a large class of n-step cutset facets that are not obtainable from previous research. We design a separation heuristic for the n-step cutset cuts and test them on both randomly generated instances by ourselves and instances from network design libraries. Our computational results show that the n-step cutset cuts are very effective in solving MMND problems. We also discuss how n-step cutset inequalities can be generalized to several variants of MMND.

Ali İrfan Mahmutoğullari, Shabbir Ahmed, Özlem Çavuş, and M. Selim Aktürk

Title: The Value of Multi-stage Stochastic Programming in Risk-averse Unit Commitment Problems

Abstract: Day-ahead scheduling of electricity generation or unit commitment (UC) is an important and challenging optimization problem in power systems. Variability in net load arising from the increasing penetration of renewable technologies have motivated study of various classes of stochastic UC models. In two-stage models, the generation schedule for the entire day is fixed while the dispatch is adapted to the uncertainty, whereas in multi-stage models the generation schedule is also allowed to dynamically adapt to the uncertainty realization. Multi-stage models provide more flexibility in the generation schedule, however, they require significantly higher computational effort than two-stage models. To justify this additional computational effort, we provide theoretical and empirical analyses of the *value of multi-stage solution* (VMS) for risk-averse multi-stage stochastic UC models. VMS measures the relative advantage of multi-stage solutions over their two-stage counterparts. Our results indicate that, for UC models, VMS increases with the level of uncertainty and decreases with the degree of risk aversion of the decision maker.

Joshua T. Margolis, Troy Long, and Scott J. Mason

Title: A mixed-integer model for the volumetric modulated arc therapy treatment planning problem

Troy Long, **Joshua T. Margolis**, Scott J. Mason

Title: A mixed-integer model for the volumetric modulated arc therapy treatment planning problem

Extended abstract: Radiation therapy is one of the main methods of treating cancer. Patients are exposed to internal or external radiation sources with the goals of 1) eradicating cancerous tissue (targets) while sparing healthy structures, or organs-at-risk (OARs), and 2) minimizing the chance of future complications. Treatment planning optimization models are used to construct a set of delivery instructions (treatment plan) to produce a desirable dose distribution when given geometric information about the patient's critical structures, which are discretized into small volumes called voxels. These models set the dose received as a function of radiotherapy machine parameters.

External beam photon treatment, which is delivered using several popular modalities, is perhaps the most common method of radiation therapy. Volumetric modulated arc therapy (VMAT) is one of the gold standards of these techniques. In VMAT, a gantry-mounted radiation source is moved along a co-planar arc around a patient fixed on a couch. While the beam moves along the arc, the shape of the beam's output is altered via a multi-leaf collimator (MLC). Because the beam is delivering radiation throughout its movement along the arc, this form of treatment is quite time-efficient. However, VMAT planning can be difficult due to the non-convex nature of the full treatment planning model. VMAT treatment planning is still an unsolved problem, although researchers can produce seemingly near-Pareto-optimal plans with respect to various dosimetric objective functions. However, there are objectives that have neither been explicitly accounted for nor considered in previous research (e.g., aperture shape/size, total beam-on-time, etc.). Commercial treatment planning systems (TPSs) have begun to include penalty functions for promoting aperture size, but these penalty parameters need to be tuned with other competing dosimetric objectives.

VMAT treatment planning has been studied in the literature previously. Solution methodologies include column generation, network algorithms, intensity modulated radiotherapy (IMRT) extensions like arc sequencing, and conformal arc therapy approaches. Each technique, though, has its own disadvantages. For instance, column generation can result in oddly shaped apertures that do not have good dosimetric qualities. Similarly, IMRT extensions such as arc sequencing have a high total beam-on time and a long total treatment time. Conformal arc approaches, moreover, do a poor job of sparing OARs. Unfortunately, a single "best in class" method does not exist in either the literature or in practice today.

Due to the continuous nature of the MLC, there is an infinite set of possible leaf positions at any given control point. Thus, the full VMAT problem is too vast to consider every possible aperture shape. In our research, we develop a mixed-integer program that creates a deliverable treatment plan with good dosimetric qualities by seeding the model with a favorable set of potential apertures. Our model works by selecting an aperture and corresponding intensity at each control point such that the deviation in the dose delivered and the dose prescribed per voxel is minimized. Apertures can only be selected if the leaves of the MLC can create the shape, given their location at the previous control point, in the time it takes the gantry to move between successive control points. We test the efficacy of our model using clinical data and compare our results to other solution methodologies previously discussed in the literature.

Akshay Gupte, Lawrence McCormick, **Hamid Nazari**

Title: Optimizing a bundle pricing problem

Extended abstract: We are interested in solving a combinatorial matrix assignment problem which appears commonly in literature as a bundle pricing problem in a multi-buyer-single-seller market. The problem has a MIQCQP formulation from the seller's perspective. Using customers' reservation prices, we wish to determine an optimal "menu" of bundles to offer, an optimal pricing scheme for the bundles, and predicting the optimal assignment of customers to bundles. The main constraint that makes this problem NP-hard in general is the requirement of bundle pricing that achieves envy-free equilibrium for the customers. Literature has many approximation results and polynomial-time algorithms under some assumptions on the reservation prices. We adopt an integer programming approach to solve this model exactly without any assumptions. Formulations and relaxations are developed for this problem, and we test these on randomly generated instances. We also compare our MIP methodology and branch-and-cut procedures to the Lagrangian relaxation techniques used in literature.

Jeffrey Larson, Sven Leyffer, **Prashant Palkar**, Stefan Wild

Title: Mixed-integer derivative-free optimization

Extended abstract: Many design and engineering applications result in optimization problems that involve so-called black-box functions as well as integer variables, resulting in mixed-integer derivative-free optimization problems (MIDFOs). MIDFOs are characterized by the fact that a single function evaluation is often computationally expensive (requiring a simulation run for example) and that derivatives of the functions can not be computed or estimated efficiently. In addition, many problems involve integer variables that are unrelaxable, meaning that one can not evaluate the functions at non-integer points. Our interest in these problems is motivated by the design of concentrating solar-power plants. Since the integer constraints are unrelaxable, one cannot apply traditional branch and bound approaches. In particular, constructing a surrogate model of the continuous relaxation of the problem and then branching, is not viable. Similarly, other traditional mixed-integer programming techniques such as Benders decomposition or outer approximation cannot be used, because gradients of the objective function are unavailable. Other derivative-free approaches used for handling integer decision variables do not guarantee convergence to a global minimum.

We present a new method for derivative-free optimization over bounded unrelaxable integer variables and prove global convergence under idealistic convexity assumption on the objective function. To the best of our knowledge, this is the first globally convergent method for unrelaxable integers apart from complete enumeration. Our method constructs secant linear mappings, that interpolate the objective function at previously evaluated points. We show that these secant functions underestimate the objective in certain truncated polyhedral cones in the feasible region. We derive 'conditional' cuts using these secants, which collectively yield a nonconvex piecewise linear underestimator model of the objective function over the entire feasible region. We show that these cuts can be formulated as a standard mixed-integer linear program (MILP). Unfortunately, solving a sequence of such MILP models to obtain new iterates turns out to be prohibitively expensive, even with state-of-the-art MILP solvers. We develop an alternative approach that is computationally tractable, and provide some early numerical experience with our new method.

Amitabh Basu, **Sriram Sankaranarayanan**

Title: Can cut generating functions be good and efficient?

Extended abstract: Theory for using multiple rows of the simplex tableaux simultaneously to obtain valid inequalities for general mixed-integer programs is now at a reasonably mature stage. The standard approach for generating such valid inequalities is to consider the so-called *Corner Relaxations* and use a structured family of subadditive functions called the *minimal valid functions*. A particularly well-studied subclass of cut generating functions that arise as the gauge functions of *lattice-free polytopes*. This lends a nice geometric view on these valid inequalities.

However, working with arbitrary lattice-free polytopes pose various difficulties. To start with, there is no canonical way to represent or parametrize the family of lattice-free polytopes. Moreover, given a lattice-free polytope, a stronger inequality can be obtained using the so-called *trivial lifting* compared to the simple gauge function. However, the trivial lifting can be hard to compute even in small dimensions. Finally, even if the trivial lifting can be computed, it could be arbitrarily poorer than the best "lifting" that gives the strongest possible cut obtainable from this lattice-free polytope. In fact, one of the results in our paper rigorously shows that the trivial lifting can be arbitrarily worse compared to the strongest "lifting".

THEORETICAL GUARANTEES: We identify a family of lattice-free convex sets which we call as generalized cross-polytopes, which have the following properties.

- The family can be parameterized by two vectors of size equal to the dimension of the cross-polytope.
- We also show that the family is rich enough that the closure of the cuts obtained through trivial liftings of the family gives a mathematically precise approximation of the closure obtained from all lattice-free convex sets.
- We provide algorithms to compute the trivial liftings of this family efficiently in arbitrary dimensions. The time to compute a gauge and the cut coefficient are given by at most $O(2^n)$ and $O(n2^n)$ elementary operations respectively. However we also identify sub families where the gauge and the lifting can both be computed with $O(n)$ elementary operations.

COMPUTATIONAL TEST RESULTS: Backed by the theoretical guarantees, we tested the method on about 13,000 random instances of problems. Roughly half of them were pure-integer problems and half were mixed-integer problems. The cuts to be compared were added to the root node of the branch-and-bound framework. Comparison were done between the following: Bounds obtained by adding (i) all GMI cuts (ii) five cuts derived from randomly chosen generalized cross-polytopes (GX-cuts) and all GMI cuts. This framework was adopted to quantify the push that the new family of cuts can give beyond the GMI cuts. We define the increase in performance as follows. If LP refers to the objective value of the linear program relaxation, GMI refers to the objective after adding the GMI cuts and Cut be the objective after adding the cuts obtained from generalized cross-polytopes along with GMI, we define the gain, $\beta = \frac{\mathsf{Cut}-\mathsf{GMI}}{\mathsf{GMI}-\mathsf{LP}}$.

Santanu S. Dey, **Asteroide Santana**, Yang Wang

Title: New SOCP relaxation and branching rule for bipartite bilinear programs

Extended abstract: A bipartite bilinear program (BBP) is a quadratically constrained quadratic program where the variables can be partitioned into two sets such that fixing the variables in any one of the sets results in a linear program. In other words, BBP is an optimization problem of the following form:

$$\begin{array}{rl} \min & x^\top Q_0 y + d_1^\top x + d_2^\top y \\ \text{s.t.} & x^\top Q_k y + a_k^\top x + b_k^\top y + c_k = 0,\ k \in \{1, \ldots, m\} \\ & (x, y) \in [0,1]^{n_1+n_2}, \end{array} \tag{1}$$

where $n_1, n_2 \in \mathbb{Z}_+,\ Q_0, Q_k \in \mathbb{R}^{n_1 \times n_2},\ d_1, a_k \in \mathbb{R}^{n_1},\ d_2, b_k \in \mathbb{R}^{n_2},\ c_k \in \mathbb{R}$, for $k \in \{1, \ldots, m\}$.

We show that the convex hull of the set defined by a single bilinear equation is second order cone (SOCP) representable in the extended space, where we have introduced new variables w_{ij} for $x_i y_j$, for all i, j such that the bilinear term is present in the model.

The SOCP relaxation for the feasible region of the general BBP (1) that we propose is the intersection of the convex hull of each of the constraints of (1):

$$S^{SOCP} := \bigcap_{k=1}^{m} \text{conv}(S_k),$$

where $S_k = \{(x, y, w) \in [0,1]^{n_1 \times n_2 \times |E|} \mid x^\top Q_k y + a_k^\top x + b_k^\top y + c_k = 0, w_{ij} = x_i y_j\ \forall (i,j) \in E\}$ and E is the set of edges $(i, j) \in V_1 \times V_2$ for which $x_i y_j$ is present in the model (and not just one row). We then show that S^{SOCP} is stronger than the standard semi-definite programming (SDP) relaxation intersected with the boolean quadratic polytope

Inspired by the convex relaxation described above, we propose a new rule for partitioning the domain of a given variable in order to produce two branches. The main ideas behind our new proposed branching rule is as follows. Suppose we have decided to branch on the variable x_1. The convex hull of the one constraint set is obtained by taking the convex hull of union of sets obtained by fixing all but two (or one) variables. We then examine all such two-variable sets involving x_1 obtained from each of the constraints. For each of these sets, there is an ideal point to divide the range of x_1 so that the sum of the volume of the two convex hulls of the two-dimensional sets corresponding to the two resulting branches is minimized. We present a heuristic to find this ideal point. We collect all such ideal points corresponding to all the two-dimensional sets involving x_1. Then we present another heuristic to select one of these points (based on corresponding volume reduction) to finally partition the domain of x_1. We also use similar arguments to propose a new variable selection rule.

We tested our relaxation on instances of the so called *finite element model updating problem*, which is a fundamental methodological problem in structural engineering. Our computational experiments on this problem class show that the new branching rule together with an polyhedral outer approximation of the SOCP relaxation described above outperforms the state-of-the-art commercial global solver BARON 15.6.5 in obtaining dual bounds. In fact, the average duality gap remaining after running our method is 13%, whereas BARON's is 68%.

Thiago Serra, Christian Tjandraatmadja, Srikumar Ramalingam

Title: Bounding and Counting Linear Regions of Deep Neural Networks

Extended abstract: We investigate the complexity of deep neural networks (DNN) that represent piecewise linear (PWL) functions. In particular, we study the number of linear regions, i.e. pieces, that a PWL function represented by a DNN can attain, both theoretically and empirically. We present (i) tighter upper and lower bounds for the maximum number of linear regions on rectifier networks, which are exact for inputs of dimension one; (ii) a first upper bound for multi-layer maxout networks; and (iii) a first method to perform exact enumeration or counting of the number of regions by modeling the DNN with a mixed-integer linear formulation. These bounds come from leveraging the dimension of the space defining each linear region. The results also indicate that a deep rectifier network can only have more linear regions than any shallow counterpart with same number of neurons if that number exceeds the dimension of the input.

Ashutosh Mahajan, **Meenarli Sharma**

Title: Automatic reformulation techniques for mixed-integer nonlinear programs (MINLPs)

Extended abstract: One can exploit specific structures in a convex MINLP whose convex hull description can be easily obtained. This typically strengthens the relaxation of the original problem. We implement two such reformulation techniques. First is Perspective Reformulation (PR) proposed by Frangioni and Gentile (2006), and Günlük and Linderoth (2010) which replaces two convex sets (also called 'on-off' sets in this context), obtained for different values of a binary variable, by their convex hull. The convex-hull description contains perspective of the function defining the on-off sets. Second technique exploits group separability of nonlinear function in the problem. It has been shown in literature that outer-approximations of convex subexpressions of a separable function yield tighter relaxation than the one obtained by outer-approximating the original function. Motivated by the effectiveness of these reformulation techniques, we automate (1) identification of two different structures amenable to PR (2) generation of 'perspective cuts' to solve PR problems (3) reformulation of problem based on group separability of nonlinear function. We present numerical results on instances from MINLPLib2 (www.minlplib.org): synthesis problems from chemical industry that have structures amenable to PR, and facility location and layout problems that have separable functions.

Morover, in some cases reformulation based on function separability induces structure amenable to PR. One such application is uncapacitated facility location problem. We present our results using a branch-and-cut framework, LP/NLP based branch-and-bound algorithm with automatic (1) PR and generation of perspective cuts (2) function separability exploitation (3) separability exploitation inducing structure amenable to PR. All our implementations are in MINOTAUR, an open-source MINLP framework by Mahajan et al. (2017).

Sahar Tahernejad, Ted K. Ralphs

Title: Valid Inequalities for Mixed Integer Bilevel Linear Optimization Problems

Extended abstract: We provide an analysis of the performance of the known classes of valid inequalities for *mixed integer bilevel linear optimization problems* (MIBLPs) within the `MibS` open source solver. We further describe an algorithm for generating inequalities that support the convex hull of the feasible region and use this procedure as a baseline for comparing the strength of various other classes.

We let $x \in X$ and $y \in Y$ represent the set of variables controlled by the first- and second-level *decision makers* (DMs) respectively, where $X = \mathbb{Z}_+^{r_1} \times \mathbb{R}_+^{n_1 - r_1}$ and $Y = \mathbb{Z}_+^{r_2} \times \mathbb{R}_+^{n_2 - r_2}$. The general form of an MIBLP is then given by

$$\min \left\{ cx + d^1 y \;\middle|\; x \in X, y \in \mathcal{P}_1(x) \cap \mathcal{P}_2(x) \cap Y, d^2 y \leq \phi(b^2 - A^2 x) \right\}, \tag{MIBLP}$$

where

$$\begin{aligned} \mathcal{P}_1(x) &= \left\{ y \in \mathbb{R}_+^{n_2} \;\middle|\; G^1 y \geq b^1 - A^1 x \right\}, \\ \mathcal{P}_2(x) &= \left\{ y \in \mathbb{R}_+^{n_2} \;\middle|\; G^2 y \geq b^2 - A^2 x \right\} \text{ and} \\ \phi(\beta) &= \min \left\{ d^2 y \;\middle|\; G^2 y \geq \beta, y \in Y \right\} \quad \forall \beta \in \mathbb{R}^{m_2}. \end{aligned} \tag{VF}$$

With respect to a given $x \in X$, the *rational reaction set* is defined as

$$\mathcal{R}(x) = \operatorname{argmin} \left\{ d^2 y \;\middle|\; y \in \mathcal{P}_2(x) \cap Y \right\}$$

and contains $y \in Y$ such that y is an optimal solution to the second-level problem parameterized on x. The *bilevel feasible region* is defined as

$$\mathcal{F} = \{(x, y) \in X \times Y \mid y \in \mathcal{P}_1(x) \cap \mathcal{R}(x)\}.$$

Dropping the optimality constraint with respect to the second-level problem, we obtain the relaxed feasible region

$$\mathcal{S} = \{(x, y) \in X \times Y \mid y \in \mathcal{P}_1(x) \cap \mathcal{P}_2(x)\}.$$

Further dropping the integrality constraints, we get

$$\mathcal{P} = \left\{ (x, y) \in \mathbb{R}_+^{n_1 \times n_2} \;\middle|\; y \in \mathcal{P}_1(x) \cap \mathcal{P}_2(x) \right\}.$$

We have $\mathcal{F} \subseteq \mathcal{S} \subseteq \mathcal{P}$.

We consider three classes of inequalities for MIBLPs based on what types of solutions they remove.

1. Feasibility cuts: Inequalities that are violated by a fractional extreme point $(\bar{x}, \bar{y})$ of $\mathcal{P}$ (or the feasible region of the current relaxation), but are valid for $\mathcal{S}$.

2. Optimality cuts: Inequalities that are violated by an integral extreme point $(\bar{x}, \bar{y})$ of $\mathcal{P}$ (or the feasible region of the current relaxation), but are valid for $\mathcal{F}$.

3. Projected optimality cuts: Inequalities violated by non-improving bilevel feasible solutions, but valid for $\text{conv}\left(\{(x, y) \in \mathcal{F} \mid cx + d^1 y < U\}\right)$, where U represents the incumbent.

We describe inequalities in each of these classes and compare their performance based on their impact on the branch-and-cut algorithm described implemented in `MibS` , as well as on comparison to the aforementioned method for generating strong supporting inequalities.

Sebastian Pokutta, Mohit Singh, **Alfredo Torrico**.

Title: Submodular maximization under matroid constraints: a case for robustness.

Extended abstract:

Over the last years, submodular maximization has caught significant attention due to its applicability in numerous real-world applications, particularly those related to constrained subset selection problems such as influence in social networks, clustering, non-parametric learning, sensor placement, to name a few. A simple example is the problem of selecting a subset of patients that is the most informative in a group of people with certain illness. Submodularity reflects the decreasing marginal gain in the information acquired via bio-medical observations when choosing more patients. Formally, a set function $g : 2^V \to \mathcal{R}_+$ is *submodular* if and only if for any $e \in V$ and $A \subseteq B \subseteq V \backslash \{e\}$, $g_A(e) \geq g_B(e)$, where $g_A(e) := g(A+e) - g(A)$ and $A+e := A \cup \{e\}$. Also, we say that g is *monotone* if for any $A \subseteq B \subseteq V$, we have $g(A) \leq g(B)$. However, there are a few reasons that motivated constrained robust submodular optimization. For instance, when running medical tests on patients, any malfunction in the procedure will lead to imprecise observations, which translates in unstable models. Thus, the goal is to obtain solutions that are robust to these perturbations. This can be achieved by optimizing against (the minimum of) several submodular functions.

In this work, we consider robust submodular maximization with matroid constraints. Formally, we are given a collection of k monotone submodular functions defined on the same ground set and a matroid constraint. The goal is to obtain a feasible subset that maximizes the minimum of the k functions. It is known that this problem is NP-hard to approximate within any polynomial factor, thus previous works have introduced bi-criteria algorithms to attack the problem. However, in real-world applications these algorithms are inefficient due to the number of function calls. Therefore, we give an efficient bi-criteria approximation algorithm that outputs a small family of feasible sets whose union has (nearly) optimal objective value. This algorithm theoretically performs less function calls than previous works at cost of adding more elements to the final solution. We also provide significant implementation improvements showing that our algorithm outperforms the algorithms in the existing literature. Finally, we provide a computational study to show these improvements.

James Luedtke, **Eli Towle**

Title: Intersection disjunctions for reverse convex sets

Extended abstract: We consider valid inequalities for sets of the form $P \setminus C$, where P is a polyhedron and C is an open convex set. This is a general set structure arising in the context of mixed integer nonlinear programming (MINLP), where P is the linear programming (LP) relaxation of a MINLP feasible region, and C is constructed in such a way to not contain any solutions feasible to MINLP. That is, C is defined by reverse convex inequality. We call sets of the form $P \setminus C$ *reverse convex sets*. We are interested in cases where the convex set C is either non-polyhedral, or is a polyhedron defined by a large number of inequalities. Reverse convex sets have applications to difference of convex functions (Tuy, 1986), bilevel optimization (Fischetti et al., 2016), mixed-integer quadratically constrained programming (Saxena et al., 2010), and polynomial optimization (Bienstock et al., 2016).

Intersection cuts are a framework for constructing valid inequalities for $P \setminus C$. Intersection cuts are generated from a basic solution within C. Consider the polyhedron $P = \{x \in \mathbb{R}^n_+ \colon Ax = b\}$, a basis B, nonbasic variables $N = \{1, \ldots, n\} \setminus B$, and a basic solution $\bar{x} \in C$. A relaxation of P is as follows:

$$P^B = \Big\{x \in \mathbb{R}^n \colon x_i = \bar{b}_i - \sum_{j \in N} \bar{a}_{ij} x_j, i \in B,\ x_j \geq 0, j \in N\Big\}.$$

Let $\{\bar{r}^j \colon j \in N\}$ be the $|N|$ linearly independent extreme rays of P^B. For $j \in N$, let

$$\beta_j \coloneqq \sup\{\lambda \geq 0 \colon \bar{x} + \lambda \bar{r}^j \in C\}.$$

We use the convention $1/+\infty \coloneqq 0$. The standard intersection cut $\sum_{j \in N} x_j/\beta_j \geq 1$ is valid for $P^B \setminus C \supseteq P \setminus C$. Intersection cuts can be generated from both feasible and infeasible basic solutions.

Intersection cuts assume the generating basic solution lies within C. Our contribution is a disjunctive framework for generating valid inequalities for $P \setminus C$ from basic solutions *outside* of the set $\operatorname{cl} C$. We first consider a basic solution $\bar{x} \notin \operatorname{cl} C$. Suppose each extreme ray of P^B intersects the convex set C. Let

$$\alpha_j \coloneqq \inf\{\lambda \geq 0 \colon \bar{x} + \lambda \bar{r}^j \in C\}.$$

Theorem 1. *For every $x \in P^B \setminus C$, either*

$$\sum_{j \in N} \frac{x_j}{\alpha_j} \leq 1 \text{ or } \sum_{j \in N} \frac{x_j}{\beta_j} \geq 1. \tag{2}$$

Theorem 1 provides a two-way disjunction for $P \setminus C$ when each extreme ray of P^B intersects the convex set C.

Our second contribution extends the two-way disjunction of Theorem 1 to a more general multi-term disjunction by considering the recession directions of C. Standard intersection cuts do not consider $\operatorname{recc} C$ in their derivation, beyond the knowledge that $\bar{r}^j \in \operatorname{recc} C$ for all $j \in N$ satisfying $\beta_j = +\infty$. We present a polyhedral relaxation for each term of the disjunction that leverages $\operatorname{recc} C$. In some cases, these inequalities are facet-defining for their respective disjunctive terms. Given these polyhedral relaxations, the multi-term disjunction can be used in a disjunctive program to generate valid inequalities for $\operatorname{cl}\operatorname{conv}(P \setminus C)$.

Hamidreza Validi, Austin Buchanan

Title: A Note on "A linear-size zero-one programming model for the minimum spanning tree problem in planar graphs"

Extended abstract: In 2002, Williams (*Networks*, 2002) proposed an extended formulation for spanning trees of a planar graph. The formulation is remarkably small (using only linearly many variables and constraints) and remarkably strong (defining an integral polytope). This shows that the spanning tree polytope of a simple planar graph $G = (V, E)$ has extension complexity $O(n)$, where $n = |V|$ is the number of its vertices.

This result is frequently mentioned as an exemplary example of the power of extended formulations, and is a key ingredient in several subsequent extended formulations. Indeed, a recent paper by Fiorini et al. (*Discrete & Computational Geometry*, 2017), which gives size $O(g^{1/2}n^{3/2} + g^{3/2}n^{1/2})$ extended formulations for spanning trees in graphs of genus g, depends crucially on Williams' result. The spanning tree formulation and a subsequent formulation for vertex-induced connectivity by Williams (*Geographical Analysis*, 2002) are also used computationally.

We show that Williams' spanning tree formulation is incorrect as stated. Specifically, we construct a binary feasible solution to Williams' spanning tree formulation that does not represent a spanning tree. Fortunately, a small tweak corrects the formulation. The change is to restrict the choice of the root vertices in the primal and dual spanning trees, whereas Williams explicitly allowed them to be chosen arbitrarily. That this restriction on the roots results in a correct formulation was first observed by Pashkovich in his dissertation (Otto-von-Guericke-Universität Magdeburg, 2012), but the result and its proof have not appeared outside of his dissertation, nor does the dissertation point out the error with Williams' original formulation. Additionally, we prove the converse statement: if Pashkovich's Root Rule is not followed, then Williams' formulation will allow a solution that is not actually a spanning tree.

Then, we give a counterexample to a subsequent formulation from Williams for vertex-induced connected subgraphs of planar graphs, which he introduced in the context of acquiring contiguous parcels of land (Williams, *Geographical Analysis*, 2002). That this second formulation is incorrect as stated is perhaps unsurprising given that it relies on the correctness of the spanning tree formulation. Fortunately, the same Root Rule patches it.

Jiawei Wang, Matthias Köppe

Title: Characterization and Approximation of Strong General Dual-Feasible Functions

Extended abstract: Dual-feasible functions (DFFs) are a fascinating family of functions, which have been used in several combinatorial minimization problems to generate lower bounds efficiently. DFFs are in the scope of superadditive duality theory, and superadditive and nondecreasing DFFs can provide valid inequalities for general integer linear programs. A function ϕ : $[0, 1] \to [0, 1]$ is called a (valid) classical DFF, if for any finite list of real numbers $x_i \in [0, 1]$, $i \in I$, it holds that, $\sum_{i \in I} x_i \leq 1 \Rightarrow \sum_{i \in I} \phi(x_i) \leq 1$. A function $\phi : \mathbb{R} \to \mathbb{R}$ is called a (valid) general DFF, if the same inequality holds for $x_i \in \mathbb{R}$.

A hierarchy on the set of valid DFFs can be defined to indicate the strength of the corresponding valid inequalities and lower bounds. The point-wise non-dominated DFFs are called *maximal*, and a maximal DFF is said to be *extreme* if it cannot be written as a convex combination of two distinct maximal DFFs. We give a full characterization of maximal general DFFs in terms of the "generalized symmetry condition" as follows.

A function $\phi \colon \mathbb{R} \to \mathbb{R}$ is a maximal general DFF if and only if the following conditions hold: (i) $\phi(0) = 0$. (ii) ϕ is superadditive. (iii) $\phi(x) \geq 0$ for all $x \in \mathbb{R}_+$. (iv) ϕ satisfies the generalized symmetry condition $\phi(r) = \inf_k \{\frac{1}{k}(1 - \phi(1 - kr)) : k \in \mathbb{Z}_+\}$.

Parallel to the restricted minimal and strongly minimal functions in the Yıldız–Cornuéjols model, "restricted maximal" and "strongly maximal" general DFFs can be defined by strengthening the notion of maximality. We also give the characterizations of restricted and strongly maximal general DFFs.

Cut-generating functions play an essential role in generating valid inequalities which cut off the current fractional basic solution in a simplex-based cutting plane procedure. We study the relation between general DFFs and certain cut-generating functions. Yıldız and Cornuéjols introduced a model of cut-generating functions generalizing the Gomory–Johnson setting. In the single-row Gomory–Johnson model, the basic variables are in $\mathbb{Z}$. Yıldız and Cornuéjols considered the basic variables to be in any set $S \subset \mathbb{R}$. We connect general DFFs to the Yıldız–Cornuéjols model with various sets S. General DFFs generate valid inequalities for the Yıldız–Cornuéjols model with $S = (-\infty, 0]$. Jeroslow, Blair and Bachem studied the valid inequalities for a classic model which fits in the Yıldız–Cornuéjols setting where $S = \{0\}$. We introduce a conversion between general DFFs and a family of cut-generating functions which generate valid inequalities for the model where $S = \{0\}$. The main idea is that valid inequalities generated by cut-generating functions for $S = \{0\}$ can be lifted to valid inequalities generated by general DFFs for the relaxation $S = (-\infty, 0]$.

The extremality of general DFFs is another focus of our results. Inspired by the famous Gomory–Johnson's 2-slope theorem, we prove the following 2-slope theorem for general DFFs. We show that any continuous piecewise linear maximal general DFF with only 2 slope values, one of which is 0, is extreme. An important application of the 2-slope theorem is our approximation theorem, which indicates that continuous extreme (2-slope) general DFFs are dense in the (convex) set of continuous restricted maximal general DFFs.

Santanu S. Dey, Rahul Mazumder, **Guanyi Wang**

Title: Convex integer programming model to obtain dual bounds for sparse PCA

Extended abstract: Principal component analysis (PCA) is one of the most widely used dimensionality reduction method in statistics and data science. In many applications, for additional interpretability, it is desirable to require cardinality constraint on the principal directions, resulting in a problem known as the sparse principal component analysis (SPCA) $\arg\max_{\|x\|_2 \leq 1, \|x\|_0 \leq k} x^\top Ax$. Unlike the PCA problem, the SPCA problem is NP-hard and there is no polynomial time algorithm to find a solution within a constant approximation ratio under some reasonable conjectures. Most conventional methods for solving SPCA are heuristics that use spectral techniques, alternating methods and the related power methods. Empirical results show that these heuristic methods work "well" conditioning on some strong and often unverifiable assumptions on the problem data which has no guarantees in general cases. Our ultimate goal is a framework that allows the computation of good solutions to SPCA problem with associated certificates of optimality, via dual bounds, which make no restrictive/unverifiable assumptions on the data. However, the only dual bounds presented in the literature are based on computationally expensive semi-definite programming (SDP) relaxations. In this paper, we present a framework that involves solving convex integer programs (IP) to produce dual bounds. We present worst-case results on the quality of the dual bound provided by the convex IP. In practice, we find that the dual bounds are significantly better than worst-case performance, and even better than the SDP bounds on some "real-life" instances. Moreover, solving the convex IP model using commercial IP solvers appears to scale significantly better that solving the SDP-relaxation using commercial SDP solvers. To the best of our knowledge, this is the first time, reasonable dual bounds are presented for real and artificial instances up to 1000 rows.

Site Wang, Harsha Nagarajan, Kaarthik Sundar, Hassan Hijazi, Russell Bent

Title: `POD.jl`: A Global MINLP Solver based on Adaptive Piecewise Relaxations

Extended abstract: Non-convex, mixed-integer nonlinear programs (MINLPs) are hard optimization problems to solve to global optimality. State-of-the-art global solvers, such as BARON, Couenne, and SCIP, often handle MINLPs using the spatial branch-&-bound approach in combination with various other tools. However, there has recently been a great deal of interest in MILP-based approaches for solving MINLPs that are based on piecewise convex relaxations. In this work, we present a MILP-based global solver for MINLPs, named POD, where the name represents the three key solution techniques to solve MINLPs with nonlinearities due to *multilinear functions*: Piecewise convex relaxation (P), Outer-approximation (O), and Dynamic partitioning (D).

At its core, POD is an iterative algorithm for solving MINLPs. Each iteration of the algorithm (1) partitions the variable domains around the previous iteration's solution using a dynamic partitioning scheme, (2) derives a piecewise convex relaxation of the MINLP using the partitions of (1), (3) solves the relaxation using outer-approximation, and (4) recovers a feasible solution to the MINLP based on the reduced variable domain implied by the relaxed solution. At each iteration, the solution to the piecewise convex relaxation is a guaranteed a bound on the optimal objective value and the bound is guaranteed to improve at subsequent iterations. The algorithm makes two key novel contributions: 1) dynamic variable partitioning scheme and 2) tight, compact piecewise convex relaxations that vary with the type of nonlinearity of the MINLP. In comparison to state-of-the-art global solvers, we demonstrate the effectiveness of POD on extensive benchmark instances several standard MINLP libraries. Among all tested instances, POD was able to outperform other global solvers and was able to improve best known optimality gap on a few open MINLP problems.

POD modularizes the architecture of the iterative algorithm by isolating the nonlinearities in the MINLP. This is done by performing expression parsing on each term in the objective function and constraints of the problem. Once the nonlinearities are isolated, a piecewise convex relaxation is derived for each nonlinear term using the variable partitions for all the variables involved in the nonlinear term. The software architecture of the piecewise convex relaxations supports user callback functionality that can utilize user-defined piecewise convex relaxations in place of the defaults. Several other features of a typical global optimization solver like pre-processing, optimality based bound-tightening, bound-propagation techniques etc. are also implemented with user-callback functionality in POD. To have both flexible usage and computational performance, POD (https://github.com/lanl-ansi/POD.jl) is implemented in the `Julia` programming language with the support of modeling and solver interfaces (i.e., `JuMP.jl` and `MathProgBase.jl`, respectively). POD utilizes `Julia`'s "first-class functions" feature to allow users to easily expand and develop solution schemes for dedicated applications and other academic research projects.

Qie He, **Zeyang Wu**

Title: Optimal switching sequence for switched linear systems

Extended abstract: We study a discrete optimization problem over a dynamical system that consists of several linear subsystems. We focus on the following discrete-time switched linear system:

$$x(k+1) = T_k x(k), \qquad T_k \in \Sigma,\ k = 0, 1, \ldots, K-1,$$

where the initial vector $x(0)$ is a given n-dimensional real vector a and the set Σ contains m $n \times n$ real matrices, each of which describes the dynamics of a linear subsystem. We aim to find a sequence of K matrices, each chosen from Σ, to maximize a convex function over $x(K)$. This problem has many practical applications and is closely connected to several fundamental problems in control and optimization. We give three examples here. Firstly, it can model the process of mitigating antibiotic resistance, in which each component of $x(k)$ represents the percentage of a genotype of an enzyme produced by bacteria, each matrix represents the mutation rates between different genotypes under certain antibiotic, and the goal is to maximize the population of the wild type after K periods. Secondly, it generalizes the matrix K-mortality problem which asks whether the zero matrix can be expressed as a product of K matrices in Σ (duplication allowed). Thirdly, it is closely related to computing the joint spectral radius of a set of matrices which generalizes the spectral radius of one matrix and has found wide applications in a variety of seemingly irrelevant fields, such as wavelet functions, switching systems, constrained coding, and network security management.

Our contributions. We show that this problem is NP-hard for a pair of stochastic matrices or binary matrices. We propose a polynomial-time exact algorithm for the problem when all input data are rational and the given set of matrices Σ has the oligo-vertex property, a new concept we introduce below. Let $P_k(\Sigma, a)$ be the convex hull of all possible values of $x(k)$, i.e.,

$$P_k(\Sigma, a) := \text{conv}(\{x(k) \mid x(k) = T_{k-1} \cdots T_0 a, T_j \in \Sigma, j = 0, \ldots, k-1\}).$$

Let $N_k(\Sigma, a)$ be the number of extreme point of $P_k(\Sigma, a)$ and $N_k(\Sigma) = \sup_{a \in \mathbb{R}^n} \{N_k(\Sigma, a)\}$. A set of matrices Σ is said to has the **oligo-vertex property** if there exists some constant d such that $N_k(\Sigma) = O(k^d)$. The oligo-vertex property indicates that the number of extreme points of $P_k(\Sigma, a)$ grows at most polynomially in k regardless of the initial vector a, although the number of possible values of $x(k)$ grows exponentially with k in general.

We derive a set of sufficient and easy to verify conditions for a set of matrices to have the oligo-vertex property: (1) A finite set of matrices that commute; (2) A finite set of matrices containing at most one matrix with the rank higher than one; (3) A pair of 2×2 matrices sharing at least one common eigenvector; (4) A pair of 2×2 binary matrices. We also show that the oligo-vertex property is invariant under a similarity transformation. In particular, we show that $N_k(\Sigma) = O(k^4)$ when Σ is a pair of 2×2 binary matrices. Finally, we conjecture that any pair of 2×2 real matrices has the oligo-vertex property.

Subhash C. Sarin, Weijun Xie, **Jie Zhang**

Title: Multi-Product Newsvendor Problem with Customer-driven Demand Substitution: A Stochastic Integer Program Perspective

Extended abstract: This paper studies a multi-product newsvendor problem with customer-driven demand substitution, where each product, once run out of stock, can be proportionally substituted by the others. This problem has been widely studied in the literature, however, due to nonconvexity and intractability, only limited analytical properties have been reported and no efficient approaches have been proposed. This paper first completely characterizes the optimal order policy when the demand is known and reformulates this nonconvex problem as a discrete submodular maximization model. When the demand is random, we formulate the problem as a two-stage stochastic integer program, derive several necessary optimality conditions, prove the submodularity of the profit function, and also develop polynomial-time approximation algorithms and show their performance guarantees. We further propose a tight upper bound via nonanticipativity dual, which is proven to be very close to the optimal value and can yield a good-quality feasible solution under a mild condition. Our numerical investigation demonstrates effectiveness of the proposed algorithms. Moreover, several useful findings and managerial insights are revealed from a series of sensitivity analyses.

Yuri Faenza, **Xuan Zhang**

Title: Legal assignments, the EADAM algorithm

Extended abstract: We investigate the many-to-one assignment problem arising in matching markets and generalize the well-known stable matching problem. Consider the classical *school-student (or hospital-residence) assignment problem*, where each student has a strict preference order over a (possibly incomplete) set of schools, and each school in turn has a strict preference order over a (possibly incomplete) set of students. Each school also has a maximum number of students it can be assigned to. An assignment is stable if there does not exist a student and a school that prefer each other to their current assignments. It is well-known that given an instance, stable assignments always exist, and the set of stable assignments forms a distributive lattice, where one can move from the supremum of the lattice to the infimum via *rotation elimination* operations. This structural property enables many fast algorithms on a wide range of optimization / enumeration problems.

A recent paper [2] introduced the concept of *Legal assignments*, which is a superset of stable assignments. This concept corresponds to *von Neumann–Morgenstern (vNM) stable set* for cooperative games. In [2], it is shown that the set of legal assignments also has a lattice structure. In addition, the student-optimal legal assignment is Pareto-optimal for the students and it coincides with the output of Kesten's *Efficiency Adjusted Deferred Acceptance Mechanism* (*EADAM*) [1].

The goal of our work is to provide an improved structural and algorithmic understanding of those concepts. Our main results are as follows. We first show that legal assignments are *structurally* equivalent to stable assignments, and *algorithmically* not harder than stable *marriages* (i.e., one-to-one assignments). We show that, for any instance, the set of *legal* assignments coincides with the set of *stable* assignments in a sub-instance, which is obtained by removing a set $\overline{E}$ from the edges E of the original instance. This implies that legal assignments inherit all structural properties of stable assignments. We also show that $\overline{E}$ can be found in time $O(|E|)$. This enables us to extend the concept of rotations to legal assignment settings, which implies that optimization of linear functions over and enumeration of the set of legal assignments can be solved efficiently and output-efficiently, respectively.

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Yingqiu Zhang, Manish Bansal

Title: Two-Stage Stochastic (and distributionally robust) p-order Conic Mixed Integer Programs: Tight Second-stage Formulations

Extended abstract: In this research, we study two-stage stochastic p-order conic mixed integer programs (TSS-CMIPs) in which the second-stage problems have p-order conic constraints along with integer variables. The two-stage stochastic mixed integer programs (TSS-MIPs) have been extensively studied in the literature. However, most of previous research is based on linear objective function and linear constraints in both stages. As per our knowledge, TSS-CMIP has not been studied in the literature in its general form, despite the fact that it subsumes various classes of optimization problems such as TSS-MIP, TSS-QIP, two-stage robust optimization problem, TSS-MIP with chance-constraints in the second-stage, and many more. This research work is the first attempt to tackle TSS-CMIPs in its general form.

We consider TSS-CMIPs with integer variables and linear constraints in the first stage, as well as mixed integer variables and p-order conic constraints in the second stage. We first provide sufficient conditions under which the integrality constraints on the second-stage integer variables of the TSS-CMIP can be relaxed, without effecting the integrality of the optimal solution of the problem, by adding parametric (non)-linear inequalities a priori. Then, we introduce structured p-order CMIPs in the second stage of TSS-CMIPs and present convex hull description of these sets by adding paramatric nonlinear inequalities (in original space) or linear inequalities (in higher dimensional space) which satisfy the foregoing conditions. We also observed that all results for TSS-CMIPs are also applicable for the TSDR-CMIP.

We also perform computational experiments for TSS-CMIPs with $p = 2$ and structured CMIPs in the second stage (i.e. either $E^j_\omega = \mathbf{I}$ or E^j_ω is Totally Unimodular matrix). We observe that parametric (non)-linear inequalities significantly reduce the time taken to solve the extensive formulation of the TSS-CMIPs. We consider two sets of randomly generated problem instances to evaluate the effectiveness of our second-order convexification approach. The instances in the first problem set are motivated from the SIPLIB TSS-MIP instances, in particular stochastic server location and stochastic multiple binary knapsack problem instances, whereas for the second problem set, we consider instances with large number of scenarios (up to 10,000). We use CPLEX 12.70 (with its default settings) to solve the extensive formulation of the foregoing instances with and without our parametric cuts. Interestingly, we observe that after adding our parametric cuts, there is a significant reduction in the number of the second stage integer variables, thereby leading to reduction in the total solution time taken to solve these problems. For instance, CPLEX 12.70 (with its default settings) could not solve 110 out of 210 TSS-CMIP instances within a time limit of 3 hours and the allocated memory of 24 GB RAM. In contrast, after adding our parametric cuts at the root node, CPLEX could solve 107 out of these 110 (unsolvable) instances within 8.2 minutes (on average). Our parametric cuts closed the integrality gap by 31.48% (on average). These computational results show that our parametric cuts significantly reduce the total solution time for these problems.

Alberto Del Pia, Jeffrey Linderoth, **Haoran Zhu**

Title: Cutting planes for Linear programmings with Complementarity Constraints

Extended abstract: We consider globally solving a linear program with additional complementarity constraints on certain pairs of original variables. Our main method is using the general extended formulation arose from Reformulation-Linearization-Technique. We studied the structural properties of the corresponding higher dimensional set which has strong relationship with the well-known Boolean Quadric Polytope on a specific complete bipartite graph, then augmenting the extended formulation with cutting planes derived from that Boolean Quadric Polytope to achieve better computational performance of branch-and-bound based methods.